**Frequency Domain and FDTD**

When interested in results at a particular frequency, we can run a simulation using a sinusoidal excitation at the frequency of interest, or we can feed the system with an impulse (approximated using a narrow Gaussian pulse) and then convert the results to frequency domain and analyze the point and frequency of interest.

If we want to calculate the Fourier transform of the E field $E(t)$ at a frequency $f$, we can use:

$$
\tilde{E}(f) = \int_{0}^{T} E(t) \cdot e^{-j 2 \pi f t} \, dt
$$

where $T$ implies our signals are causal.

Rewriting using the finite difference approximation,

$$
\tilde{E}(f) = \sum_{k=0}^{N} \tilde{E}(k \Delta t) \cdot e^{j k f \Delta t}
$$

where $n$ is our current time step number, separating real and imaginary parts,

$$
\tilde{E}(f) = \sum_{k=0}^{N} \tilde{E}(k \Delta t) \cdot \cos(2 \pi f \Delta t k) - j \sum_{k=0}^{N} \tilde{E}(k \Delta t) \sin(2 \pi f \Delta t k)
$$

In terms of computer code, for our 1D case,

$$
\begin{align*}
\text{REAL}_-\text{PART}(m, k) &= \text{REAL}_-\text{PART}(m, k) + E_x(k) \cdot \cos(2\pi f \text{FREQ}(m) \Delta t \times n) \\
\text{IMAG}_-\text{PART}(m, k) &= \text{IMAG}_-\text{PART}(m, k) + E_x(k) \cdot \sin(2\pi f \text{FREQ}(m) \Delta t \times n)
\end{align*}
$$

where

- $k$ indicates position
- $m$ indicates index inside frequency vector FREQ()

**Amplitude and Phase** for each point and frequency are:

$$
\begin{align*}
\text{AMPMUTE}(m, k) &= \sqrt{(\text{REAL}_-\text{PART}(m, k))^2 + (\text{IMAG}_-\text{PART}(m, k))^2} \\
\text{PHASE}(m, k) &= \text{ATAN2}(\text{IMAG}_-\text{PART}(m, k), \text{REAL}_-\text{PART}(m, k))
\end{align*}
$$

or

$$
\begin{align*}
\text{PHASE}(m, k) &= \text{ATAN2}(\text{IMAG}_-\text{PART}(m, k), \text{REAL}_-\text{PART}(m, k))
\end{align*}
$$