

Assignment #2 - DUE: Monday, September 28

- Given observations

$$x[n] = A + w[n], n = 0 \dots N - 1$$

where $w[n]$ is independent identically distributed (IID) with a Laplacian PDF

$$p(w[n]) = \frac{1}{\sqrt{2}\sigma} e^{-\frac{\sqrt{2}|w[n]|}{\sigma}}$$

we are interested in estimating A . Assume σ is known. What is the minimum variance for any unbiased estimator \hat{A} ?

Hint: You might want to use the alternative form of CRLB which uses the square of the first derivative rather than the second derivative/

- Problem 3.4 from textbook.
- Problem 3.6 from textbook.
- We are given a linear model

$$x[n] = A + Bn + w[n], n = 0 \dots N - 1$$

where A and B are unknown and $w[n]$ is white Gaussian noise (WGN) with an unknown variance σ^2 . The parameter vector θ consists of three elements: A , B and σ^2 .

- Find the Fisher information matrix $\mathbf{I}(\theta)$.
- Find $\mathbf{I}^{-1}(\theta)$.

Hint: The Fisher information matrix you found in the previous step should be block diagonal, which will simplify taking its inverse. Given a block diagonal matrix of the form

$$\mathbf{M} = \begin{bmatrix} \mathbf{A} & \mathbf{0} \\ \mathbf{0} & \mathbf{B} \end{bmatrix}$$

its inverse is found as

$$\mathbf{M}^{-1} = \begin{bmatrix} \mathbf{A}^{-1} & \mathbf{0} \\ \mathbf{0} & \mathbf{B}^{-1} \end{bmatrix}$$

- Problem 4.1 from textbook.
- Consider the linear model

$$x[n] = A + Bn + w[n], n = 0 \dots N - 1$$

where $w[n]$ is WGN with known variance $\sigma^2 = 1$. For $N=10,50$ and 100 do the following:

- Generate a random sample of size N for the choice $A = -3$ and $B = 0.2$ using the MATLAB command $x = -3 + 0.2 * (0 : N - 1)' + randn(N, 1)$. Compute and record the values of the MVU estimators for A and B given this sample.

- (b) Repeat step (a) 200 times.
- (c) Compute the empirical variances of your estimators.
- (d) Compute the analytical variances using CRLB for the given value of N .

Compare the empirical and analytical variances. You don't need to include any code in your solutions.

Hint: Implementing the MVU estimators for the linear model in the vector form as discussed in Chapter 4 is easier than the form in Chapter 3. Same goes for computing the analytical bounds.