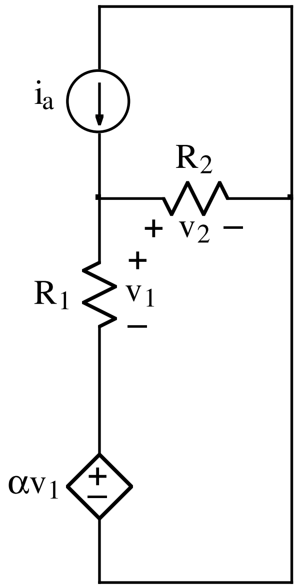
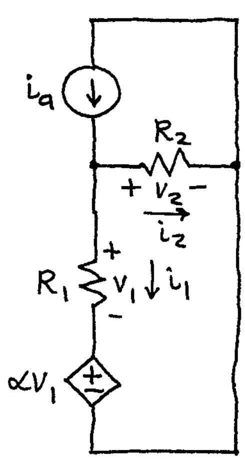
Ex:



a) Derive an expression for v_2. The expression must not contain more than the circuit parameters α, i_a, R_1, and R_2. **Note:** $\alpha > 0$.

b) Make at least one consistency check (other than a units check) on your expression. Explain the consistency check clearly.

sol'n: a) Label resistor currents.



We have a v-loop on the bottom:

$$+\alpha v_1 + v_1 - v_2 = 0\,V$$

All other loops would pass thru a current src.

We can write i-sum eq'ns for the node on the left or right but we only need one of them.

For the left node, our i-sum eq'n is

$$-i_a + i_2 + i_1 = 0\text{A}$$

We have no components in series carrying the same current, (other than a v-src and R).

Last, we write Ohm's law eq'ns for R_1 and R_2:

$$v_1 = i_1 R_1$$

$$v_2 = i_2 R_2$$

Substituting the Ohm's law expressions into the v-loop eq'n gives

$$\alpha i_1 R_1 + i_1 R_1 - i_2 R_2 = 0\text{V}$$

or $$i_1(\alpha+1)R_1 - i_2 R_2 = 0\text{V}$$

Now we solve our i-sum eq'n for i_1. (This is convenient since we will retain i_2 and be able to write an eq'n for v_2.)

$$i_1 = i_a - i_2$$

Substituting into our last v-loop eq'n, we have

$$(i_a - i_2)(\alpha+1)R_1 - i_2 R_2 = 0\text{V}$$

We simplify and solve for i_2:

$$i_a(\alpha+1)R_1 = i_2\left[(\alpha+1)R_1 + R_2\right]$$

or $$i_2 = i_a \frac{(\alpha+1)R_1}{(\alpha+1)R_1 + R_2}$$

b) Many consistency checks are possible.

ex: Set $\alpha = 0$ so dependent v-src becomes a wire. Circuit becomes i-divider.

$$i_2 = i_a \frac{R_1}{R_1 + R_2}$$

Plug $\alpha = 0$ into answer from (a):

$$i_2 = i_a \frac{(0+1)R_1}{(0+1)R_1 + R_2} = i_a \frac{R_1}{R_1 + R_2} \checkmark$$

(We get the same answer.)

ex: Set $R_2 = 0\,\Omega$ (a wire).
All current will flow thru the wire since it is the path of least resistance.

$$\therefore i_2 = i_a$$

Plug $R_2 = 0\,\Omega$ into answer from (a):

$$i_2 = i_a \frac{(\alpha+1)R_1}{(\alpha+1)R_1 + 0\,\Omega} = i_a \checkmark$$ (same answer)