

CONCEPT: Dependent sources in circuits are equivalent to resistors. To justify this claim, we demonstrate that the currents and voltages in a circuit containing a dependent source are equal to a constant times the currents and voltages obtained when the dependent source is turned off. In particular, the variable the dependent source depends on, (hereafter called the dependent source variable), is scaled by a constant compared to what its value is without the dependent source.

For example, suppose the dependent source produces a voltage αi_x . The dependent source variable is i_x . The value i_x is scaled by constant $1/(1-k\alpha)$ when the dependent source is turned on. (We explore what k is later on.)

Superposition may be used to find the value of the current in the dependent source in terms of the dependent variable, i_x . In this process, the part of the circuit where the dependent variable is measured is turned into an independent voltage source of value i_x . The dependent source is also turned into an independent source. Its value is αi_x . The two sources are turned on one at a time. Each source gives rise to current in the part of the circuit where the dependent source is located. The sum of the currents is the current used in Ohm's law to find the equivalent resistance for the dependent source.

The following step-by-step procedure yields the value of the equivalent resistance for a dependent source. The procedure is written for a voltage source of value αi_x that depends on current i_x somewhere else in the circuit, but it may be adapted to other cases by substituting "current" for "voltage" or vice versa as needed.

- i) Turn off all independent sources. A voltage source become a wire (no voltage drop) when turned off, and a current source becomes an open circuit (no current flow).
- ii) Replace the entire part of the circuit where dependent variable i_x is measured with an independent current source of value i_x . Note that all the components through which i_x flows are replaced by an independent current source, regardless of what those components are.
- iii) Treat the dependent source as an independent voltage source of value αi_x .
- iv) Use superposition to calculate the current in the dependent source in terms of i_x . First, turn on the i_x source and turn off the αi_x source. (Since the dependent

source is a voltage source, it becomes a wire when turned off, and the current being calculated is flowing in that wire.) Find the current in the dependent source (which is now a wire) in terms of i_x . We refer to this current as i_1 :

$$i_1 = k_1 i_x$$

- v) Second, turn on the αi_x voltage source and turn off the i_x source. (Since the i_x source is a current source, it becomes an open circuit when turned off.) Find the current in the dependent source in terms of i_x . We refer to this current as i_2 :

$$i_2 = k_2 \alpha i_x$$

- vi) Third, sum the two currents above and use Ohm's law to calculate the equivalent resistance of the dependent source as the voltage of the dependent source divided by the current of the dependent source:

$$R_{\text{Eq}} = \frac{\alpha i_x}{i_1 + i_2} = \frac{\alpha i_x}{k_1 i_x + k_2 \alpha i_x} = \frac{\alpha}{k_1 + k_2 \alpha}$$