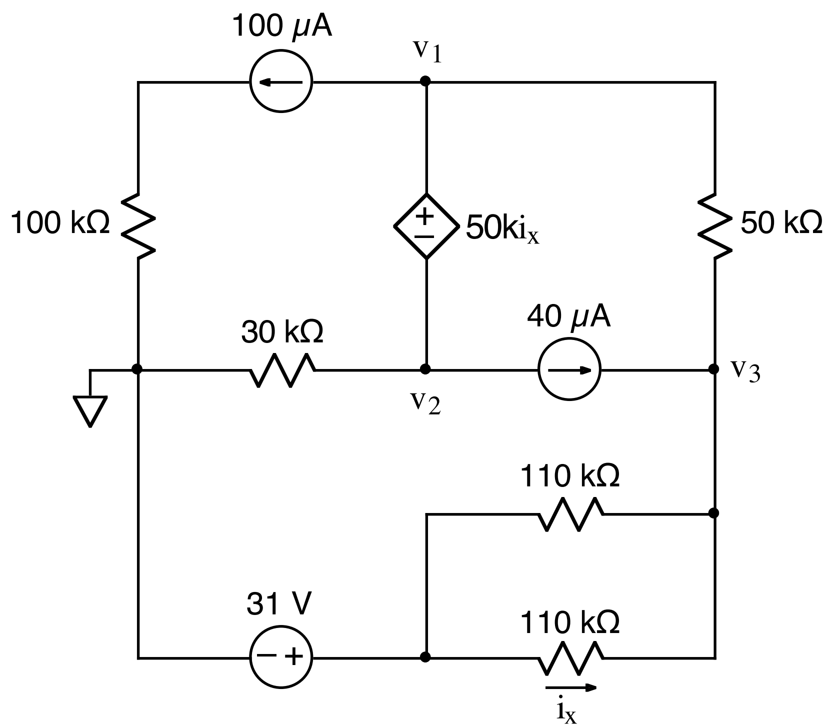
**Ex:**




Use the node-voltage method to find $v_1$, $v_2$, and $v_3$.

sol'n: We first write the variable for the dependent source, $i_x$, in terms of node V's.

$$i_x = \frac{31V - v_3}{110\,k\Omega}$$

We have a super node for $v_1$ and $v_2$ since these nodes are connected only by a V src. Thus, we sum all the currents flowing out of a bubble drawn around $v_1$, $v_2$, and the V-src between them.

$$v_1, v_2 \text{ node: } 100\mu A + \frac{v_1 - v_3}{50\,k\Omega} + \frac{v_2}{30\,k\Omega} + 40\mu A = 0A$$

We also write a voltage eq'n for $v_1$ & $v_2$:

$$v_1 = v_2 + 50\text{k}\Omega\, i_x = v_2 + 50\text{k}\Omega \left(\frac{31\text{V} - v_3}{110\text{k}\Omega}\right)$$

(Remember to use only node V's in eq'ns.)

For the $v_3$ node, we only have a current sum.

$$v_3 \text{ node: } \frac{v_3 - v_1}{50\text{k}\Omega} + -40\mu\text{A} + \frac{v_3 - 31\text{V}}{\underbrace{110\text{k}\Omega \parallel 110\text{k}\Omega}_{55\text{k}\Omega}} = 0\text{A}$$

Now we solve the 3 eq'ns. We put terms multiplying $v_1$, $v_2$, and $v_3$ on the left side and constant terms on the right side.

$$v_1 \frac{1}{50\text{k}\Omega} + v_2 \frac{1}{30\text{k}\Omega} + v_3\left(\frac{-1}{50\text{k}\Omega}\right) = -100\mu\text{A} - 40\mu\text{A}$$

$$v_1 - v_2 + v_3 \frac{50\text{k}\Omega}{110\text{k}\Omega} = 31\text{V} \cdot \frac{50\text{k}\Omega}{110\text{k}\Omega}$$

$$v_1\left(\frac{-1}{50\text{k}\Omega}\right) + v_3\left(\frac{1}{50\text{k}\Omega} + \frac{1}{55\text{k}\Omega}\right) = 40\mu\text{A} + \frac{31\text{V}}{55\text{k}\Omega}$$

We multiply both sides of the 1st eq'n by 150 kΩ to clear the denominators.

$$v_1 \cdot 3 + v_2 \cdot 5 + v_3(-3) = -140\mu\text{A} \cdot 150\text{k}\Omega = -21\text{V}$$

We multiply both sides of the 2nd eq'n by 11Ω to clear the denominators.

$$v_1(11) + v_2(-11) + v_3(5) = 31\text{V}(5) = 155\text{V}$$

We multiply the 3rd eq'n by $1100\,k\Omega = 1.1\,M\Omega$

$$v_1(-22) + v_3(22+20) = 40\cancel{\mu}A \cdot 1.1\cancel{M}\Omega + 31V(20)$$

or $v_1(-22) + v_3(42) = 44V + 620V = 664V$

Now we start eliminating variables. From the 1st eqn, we have

$$v_2 = \frac{-21V - 3v_1 + 3v_3}{5}$$

Our 2nd and 3rd eq'ns with this $v_2$ become

$$11v_1 - 11\left(-\frac{21V - 3v_1 + 3v_3}{5}\right) + 5v_3 = 155V$$

and

$$-22v_1 + 42v_3 = 664V$$

Solving the last eq'n for $v_3$ gives

$$v_3 = \frac{664V + 22v_1}{42}$$

Substituting into the 1st eq'n (and collecting terms multiplying $v_3$) yields the following eq'n:

$$\left(11 + \frac{33}{5}\right)v_1 + \left(-\frac{33}{5} + 5\right)\left(\frac{664V + 22v_1}{42}\right) = 155V - \frac{21(11)V}{5}$$

or, after multiplying both sides by 5,

$$88v_1 - 8\left(\frac{664V + 22v_1}{42}\right) = 775V - 231V = 544V.$$

Dividing both sides by 4 and moving constant term:

$$22v_1 - \frac{2(22)}{42} v_1 = 136V + \frac{2(664)}{42} V.$$

Multiplying both sides by 42 gives

$$[42(22) - 44] v_1 = 136V(42) + 2(664)V$$

or $$v_1 = \frac{5712 + 1328}{924 - 44} V = \frac{7040}{880} V = 8V$$

Then $$v_3 = \frac{664V + 22(8V)}{42} = 20V$$

and $$v_2 = \frac{-21V - 3(8V) + 3(20V)}{5} = 3V.$$

Consistency check: Calculate currents from these voltages and verify that currents sum to zero at nodes... They do!