DERIV: Ohm's law is almost always derived from basic physics with a starting assumption that the electric field inside a resistor is constant. We first investigate this assumption.

The electric field outside of an infinite, charged plate (perpendicular to the x axis) is constant and pointing normal to the plate [1, p. 29]:

$$\mathbf{E} = 2\pi\rho\hat{\mathrm{x}} \tag{1}$$

where $\rho \equiv$ surface charge density and $\hat{\mathrm{x}}$ is a unit vector in the x direction.

By Gauss's law [ref https://physics.info/law-gauss/] for a region of constant **E** field, we find zero charge density:

$$\rho = \varepsilon_0 \nabla \cdot \mathbf{E} = \varepsilon_0 \left(\frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z} \right) = 0 \tag{2}$$

where $\rho \equiv$ charge density and $\varepsilon_0 =$ permittivity of free space.

Thus, the charges that create the **E** field in a resistor must be outside (or at the ends of) the resistor. Since the endpoint connections to a resistor are too small to be approximated as infinite plates, the infinite plate assumption seems to be suspect, but it seems reasonable to assume that the process inside a resistor is uniform throughout. Certainly, we expect a steady flow of current that is constant along the length of the resistor. Otherwise, charge would accumulate somewhere, causing either an oscillation (and radio frequency transmissions) or a stable electric field that yields a constant current flow in any case.

Since the electric field is the gradient of the potential, φ, (or voltage, V) inside the resistor, the electric field must be the voltage drop across the resistor, V_0, divided by the length, l, of the resistor.

$$\mathbf{E} = \nabla \cdot \varphi \text{ or in this case } E = -\frac{d\varphi}{dx} = -\frac{dV(x)}{dx} = \frac{V_0}{l} \tag{3}$$

Having established or argued or accepted that the **E** field is constant in a resistor, the next step is to argue that electrons in a resistor will have a thermal velocity owing to temperature and a drift velocity owing to the voltage applied to the resistor. Each electron will have a random direction of motion for the thermal velocity (causing electrons to bounce around like billiard balls) plus a motion in the direction (actually, because the electron has negative charge, opposite the direction) of the electric field. The latter is called the "drift" velocity, and it accounts for the current that flows through the resistor.

To calculate the drift velocity, we consider the journey of an electron through the resistor. The electron collides with atoms or other electrons in the resistor after travelling an average time, τ, that we refer to as the "mean time between collisions". During time τ, the electron will experience constant acceleration in the direction of drift (or current flow) starting from (we assume) zero drift velocity to reach a final velocity of acceleration times time:

$$v_{\text{drift}} = \frac{\text{force}}{\text{mass}} \cdot \text{time} = -\frac{eE}{m}\tau = -\frac{eV_0}{ml}\tau \tag{4}$$

where $e \equiv$ charge of electron and $m =$ mass of electron.

It turns out that v_{drift} is much smaller than the thermal velocity, v_{T}, so we approximate that the distance travelled on average, or "mean free path", λ, is the product of v_{T} and τ:

$$\lambda = v_{\text{T}}\tau \text{ or } \tau = \frac{\lambda}{v_{\text{T}}} \tag{5}$$

Substituting for τ in (4) gives a formula for v_{drift} in terms of quantities that have identifiable physical associations.

$$v_{\text{drift}} = -\frac{eV_0\lambda}{mlv_{\text{T}}} \tag{6}$$

We expect that the mean free path, λ, will have some correspondence to the spacing between atoms in the resistor. Also, from physics, the thermal velocity is given by

$$v_{\text{T}} = \sqrt{\frac{3kT}{m}} \tag{7}$$

where $k \equiv$ Boltzmann constant and $T \equiv$ temperature in °K.

The total current flowing in the resistor is the amount of charge passing position x along the resistor in one unit of time. It follows that the total current, I, is the drift velocity times the amount of charge at a given position x along the resistor.

$$I = -eNAv_{\text{drift}} = \frac{eNAeV_0\lambda}{mlv_{\text{T}}} = V_0\left(\frac{e^2N\lambda}{mv_{\text{T}}}\right)\frac{A}{l} \tag{8}$$

where $N \equiv$ density of charge carriers (charge per volume) and
$A \equiv$ Area (cross section) of resistor.

In (8) we identify the term in parentheses as conductivity, σ.

$$\sigma = \frac{e^2N\lambda}{mv_{\text{T}}} \tag{9}$$

and, substituting in (8) we have

$$I = V_0\frac{\sigma A}{l}. \tag{10}$$

Thus, we have arrived at Ohm's law.

$$R = \frac{l}{\sigma A} \tag{11}$$

and, substituting R into (10), we have a statement of Ohm's law:

$$I = \frac{V_0}{R}. \tag{12}$$

A Note on Temperature

The appearance of temperature, T, in (7) might lead us to expect a simple variation of resistance with temperature. However, other quantities vary with temperature, too. The density of charge carriers, N, will increase with temperature, possibly in a nonlinear way, but λ will drop with temperature as vibrations of atoms becomes more pronounced, also in a possibly nonlinear way. In materials with few charge carriers, such as insulators and semiconductors, the increase in N typically wins out and resistance decreases. In conductors with large numbers of carriers, the drop in λ wins out and resistance increases.

A Comment on the Derivation of Ohm's Law

The derivation of Ohm's law ultimately depends on the flow of current in the resistor being uniform and the consequent conclusion that the physics locally will look about the same throughout the resistor. A formal, detailed justification of this uniformity, however, is hard to find in the physics literature.

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