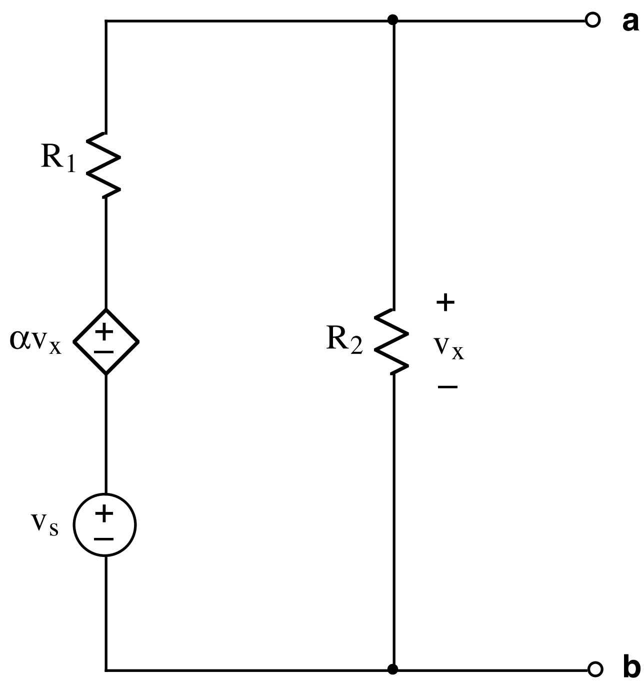
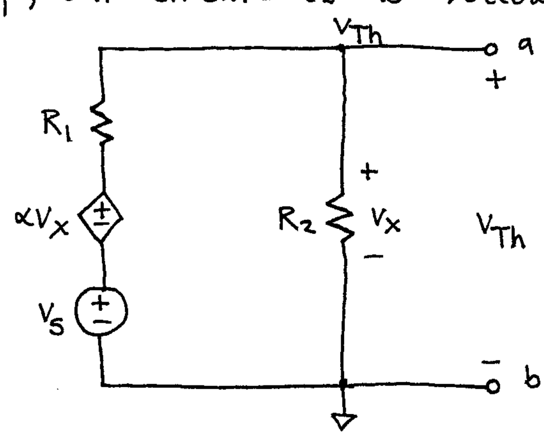
Ex:



Find the Thevenin equivalent circuit at terminals a-b. v_X must not appear in your solution. **Hint:** Use the node voltage method. **Note:** $0 < \alpha < 1$.

soln: $V_{Th} = V_{a,b}$ with nothing connected across a, b

Using the node-voltage method, with the reference at the bottom and V_{Th} at the top, our circuit is as follows:



We write v_x in terms of v_{Th}:

$$v_x = v_{Th}$$

The current summation for the v_1 node has only two terms:

$$\frac{v_{Th} - (v_s + \alpha v_{Th})}{R_1} + \frac{v_{Th}}{R_2} = 0 \text{ mA}$$

or

$$v_{Th}\left(\frac{1}{R_1} - \frac{\alpha}{R_1} + \frac{1}{R_2}\right) = \frac{v_s}{R_1}$$

Multiplying both sides by R_1 yields

$$v_{Th}\left(1 - \alpha + \frac{R_1}{R_2}\right) = v_s$$

or

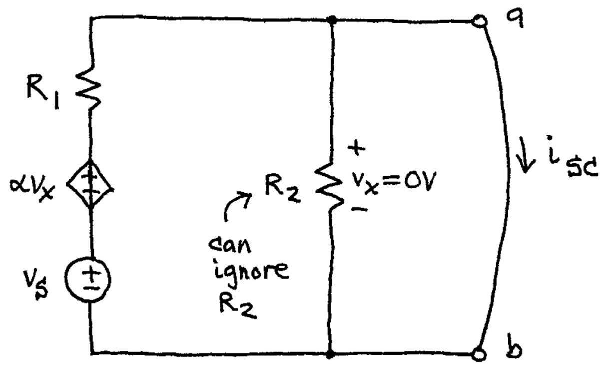
$$v_{Th} = \frac{v_s}{1 - \alpha + \frac{R_1}{R_2}} = v_s \frac{R_2}{R_2(1-\alpha) + R_1}$$

To find R_{Th}, we use $R_{Th} = \frac{v_{Th}}{i_{sc}}$

where $i_{sc} \equiv$ short circuit current flowing in a wire from a to b.

With a wire connecting a and b, $v_x = 0\text{V}$ and the dependent source becomes a wire.

Also, no current will flow thru R_2, and we may ignore R_2.



If we ignore R_2, we calculate i_{sc} from the outer loop:

$$i_{sc} = \frac{v_s}{R_1}$$

Thus, we have

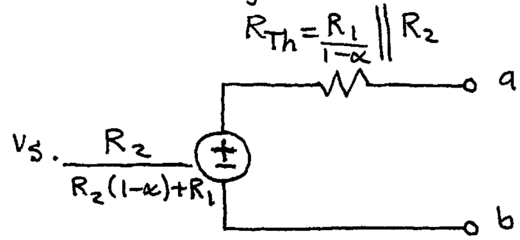
$$R_{Th} = \frac{v_{Th}}{i_{sc}} = \frac{\dfrac{v_s}{1 - \alpha + \dfrac{R_1}{R_2}}}{\dfrac{v_s}{R_1}} = \frac{R_1}{1 - \alpha + \dfrac{R_1}{R_2}}$$

Other equivalent forms for R_{Th}:

$$R_{Th} = \frac{R_1 R_2}{R_2(1-\alpha) + R_1}$$

$$R_{Th} = \frac{1}{\dfrac{1-\alpha}{R_1} + \dfrac{1}{R_2}} = \frac{R_1}{1-\alpha} \parallel R_2$$

The circuit diagram for the Thevenin equivalent:



Note: For the first part of this problem, when we calculate v_{Th}, we may use an equivalent resistance in place of the dependent source.

With a,b open circuit, the same current, i, flows in R_2 and the dependent source.

$$\therefore \quad v_X = iR_2 \qquad \text{or} \qquad i = \frac{v_X}{R_2}$$

The equivalent R for the dependent source is

$$R_{eq} = \frac{\alpha v_X}{-i} = \frac{\alpha v_X}{\frac{v_X}{R_2}} = -\alpha R_2$$

Using R_{eq} in place of αv_X allows us to obtain v_{Th} from a v-divider formula:

$$v_{Th} = v_s \frac{R_2}{R_2 - \alpha R_2 + R_1} = v_s \frac{R_2}{R_2(1-\alpha)+R_1}$$

This agrees with our node-voltage result.