DEF: Fourier series for periodic $g(x) \equiv \sum_{n=0}^{\infty} \left[a_n \sqrt{2 f_0} \cos(2\pi n f_0 t) + b_n \sqrt{2 f_0} \sin(2\pi n f_0 t) \right]$

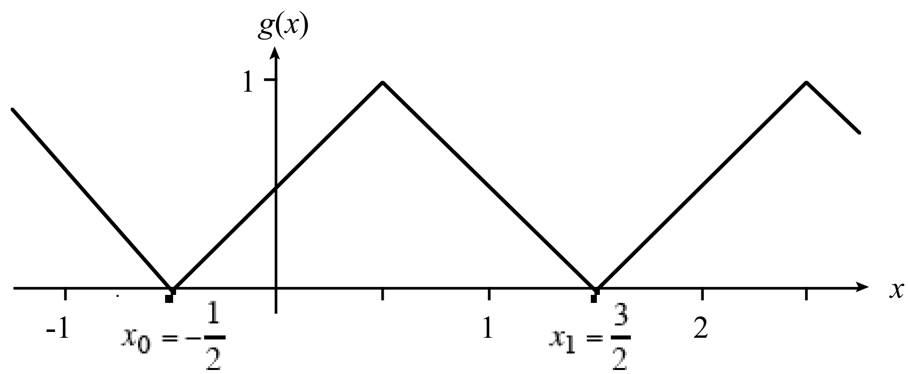
where period of $g(x)$ is periodic with period $T = \dfrac{1}{f_0}$.

NOTE: Since $\sin(2\pi n f_0 t) = 0$ for $n = 0$ and $\cos(2\pi n f_0 t) = 1$ for $n = 0$, the Fourier series for periodic $g(x)$ may also be written as

$$g(x) \equiv a_0 + \sum_{n=1}^{\infty} \left[a_n \sqrt{2 f_0} \cos(2\pi n f_0 t) + b_n \sqrt{2 f_0} \sin(2\pi n f_0 t) \right]$$

DEF: Fundamental frequency of periodic $g(x) \equiv f_0 = \dfrac{1}{T}$.

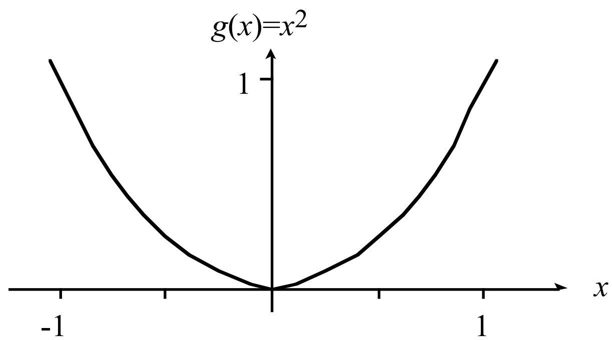
EX:



This sawtooth waveform has period $x_1 - x_0 = \dfrac{3}{2} - -\dfrac{1}{2} = 2$. Therefore, the fundamental frequency is $f_0 = \dfrac{1}{2}$. Thus,

$$g(x) \equiv a_0 + \sum_{n=1}^{\infty} \left[a_n \cos(\pi n t) + b_n \sin(\pi n t) \right]$$

EX:



The above quadratic function is not periodic. It has no Fourier series representation, (unless we extract a section from it and make side-by-side copies so it becomes periodic). We might try to use a Fourier transform representation of $g(x)$, but this approach would also fail, as the energy in the function is infinite:

$$\int_{-\infty}^{\infty} g^2(x)dx = \infty$$