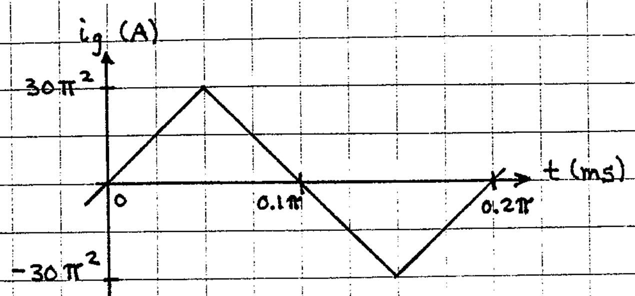
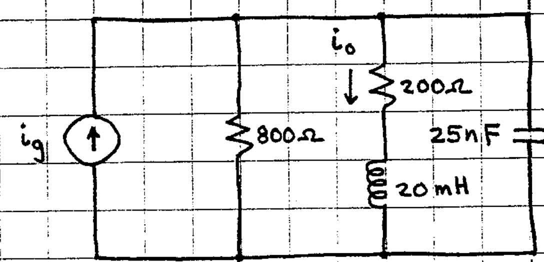




Write time-domain expression for fifth-harmonic current in expression for i_o.

ans: $i_{o5}(t) = -6.4\cos(50\text{k}t)$ A

soln: Fourier series for triangular wave such as $i_g(t)$ from inside back cover of Text:

$$i_g(t) = \frac{8A}{\pi^2}\sum_{\substack{k\text{ odd}\\k>0}}^{\infty}\left[\frac{1}{k^2}\sin\left(\frac{k\pi}{2}\right)\right]\sin k\omega_o t \quad \text{A}$$

here, $A = 30\pi^2$ and $\omega_o = \frac{2\pi}{T} = \frac{2\pi}{0.2\pi\,\text{m}} = 10\text{k rad/s}$.

We want the phasor for the 5^{th} harmonic in $i_g(t)$:

$k=5$: $\quad i_{g5}(t) = \frac{8A}{\pi^2}\frac{1}{5^2}\sin\left(\frac{5\pi}{2}\right)\sin(5\omega_o t)$ A

Now $\sin\left(\frac{5\pi}{2}\right) = \sin\left(2\pi + \frac{\pi}{2}\right) = \sin\left(\frac{\pi}{2}\right) = 1$

$$\therefore i_{g5}(t) = \frac{8A}{\pi^2}\frac{1}{5^2}\cdot 1\cdot\sin(5\omega_o t) \quad \text{A}$$

$$\text{"} \quad = \frac{8\cdot 30\pi^2}{\pi^2}\frac{1}{5^2}\sin(50\text{k}t) \quad \text{A}$$

$$\text{"} \quad = 9.6\sin(50\text{k}t) \quad \text{A}$$

Phasor: $\mathbf{I}_{g5} = 9.6\angle -90^\circ$ A $\quad$ ($P[\sin\omega t] = 1\angle -90^\circ$)

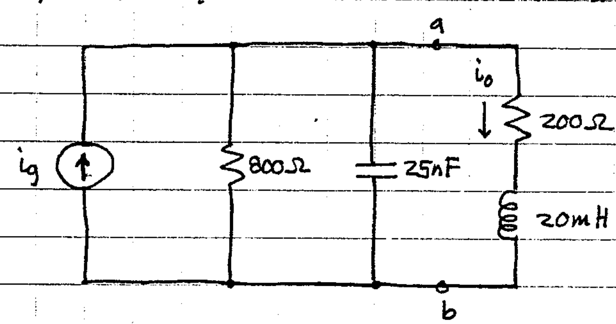
Next, we find the transfer function, $H(j\omega)$:

$$H(j\omega) \equiv \frac{\mathbb{I}_o}{\mathbb{I}_g}$$

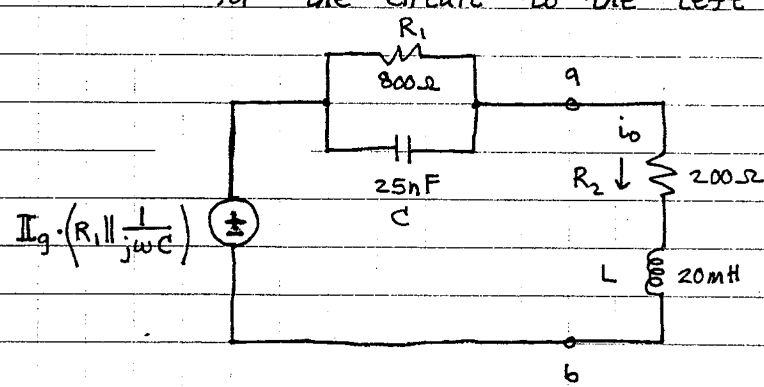
We have a current divider with 3 legs. We have a divider formula for 2 legs, but not 3. The formula for 3 legs is NOT $\dfrac{Z_1+Z_3}{Z_1+Z_2+Z_3}$ where

$Z_1 = 800\,\Omega$ $\quad Z_2 = 200\,\Omega + j\omega\, 20\,\text{mH}$ $\quad Z_3 = 1/j\omega\, 25\,\text{nF}$.

To be safe, we first rearrange the circuit by exchanging the 2nd leg (Z_2) and the 3rd leg (Z_3). This is allowed because both are across the rails:



The second step is to use a Thevenin equivalent for the circuit to the left of terminals a, b:



$$\mathbb{I}_o = \underbrace{\frac{V}{}}_{Z_{tot}} = \frac{\mathbb{I}_g \cdot \left(R_1 \| \frac{1}{j\omega C}\right)}{\left(R_1 \| \frac{1}{j\omega C}\right) + R_2 + j\omega L}$$

$$H(j\omega) = \frac{I_o}{I_g} = \frac{R_1 \| 1/j\omega C}{(R_1 \| 1/j\omega C) + R_2 + j\omega L}$$

$$R_1 \| 1/j\omega C = \frac{R_1 / j\omega C}{R_1 + 1/j\omega C} = \frac{R_1}{1 + j\omega R_1 C}$$

$$\therefore H(j\omega) = \frac{\dfrac{R_1}{1+j\omega R_1 C}}{\dfrac{R_1}{1+j\omega R_1 C} + R_2 + j\omega L} = \frac{R_1}{R_1 + (R_2 + j\omega L)(1 + j\omega R_1 C)}$$

$$= \frac{R_1}{R_1 + R_2 + j\omega(L + R_2 R_1 C) + (j\omega)^2 L R_1 C}$$

$$= \frac{1}{R_1 + R_2 + j\omega\left(\frac{L}{R_1} + R_2 C\right) + (j\omega)^2 LC}$$

$$H(j\omega) = \frac{1/LC}{\dfrac{R_1 + R_2}{R_1 L C} + j\omega\left(\dfrac{1}{R_1 C} + \dfrac{R_2}{L}\right) + (j\omega)^2}$$

5^{th} harmonic, $k = 5$: $H(j5\omega_0) = H(j50k)$.

Note: ω_0 from period of triangular wave $\neq \frac{1}{\sqrt{LC}}$.
Thus, ω_0 NOT center frequency.

$$\underset{j5\omega_0}{\overset{\|}{H(j50k)}} = \frac{1/20m\,25n}{\dfrac{800 + 200}{20m\,25n} + j50k\left(\dfrac{1}{800 \cdot 25n} + \dfrac{200}{20m}\right) + (j50k)^2}$$

$$= \frac{2\,G}{\dfrac{20\,kG}{800} + j50k\left(\underbrace{\dfrac{1}{20\mu} + 10k}_{=50k}\right) + (j50k)^2}$$

$$= \frac{2G}{\cancel{2.5G} + j\,3G - \cancel{2.5G}} = \frac{2}{j3} = -j\frac{2}{3}$$

$$H(j5\omega_0) = -j\frac{2}{3} = \frac{2}{3}\angle -90^\circ$$

We have $\mathbb{I}_{o5} = \mathbb{I}_{g5} \cdot H(j5\omega_0)$

$$" = 9.6\angle{-90^\circ} \cdot \frac{2}{3}\angle{-90^\circ} \text{ A}$$

$$\therefore \mathbb{I}_{o5} = 6.4\angle{-180^\circ} \text{ A}$$

Take inverse phasor to get $i_{o5}(t)$:

$$i_{o5}(t) = 6.4\cos(5\omega_0 t - 180^\circ) \text{ A}$$

$$= -6.4\cos(5\omega_0 t) \text{ A} \quad 180^\circ \Rightarrow \text{mult by } -1$$

$$i_{o5}(t) = -6.4\cos(50\text{k}t) \text{ A}$$