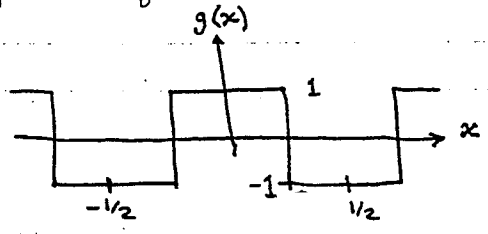
Fourier Series — Examples — Square wave

ex: Square Wave



$$g(x) = \sum_{n=0}^{\infty} a_n \sqrt{2} \cos 2\pi n x + b_n \sqrt{2} \sin 2\pi n x$$

$a_0 =$ ave value $g(x) = 0$

$$a_{n\neq 0} = (g, \sqrt{2}\cos 2\pi n x) = \int_{\text{one cycle}} g \sqrt{2} \cos 2\pi n x \, dx$$

$$= 2 \int_{\substack{\text{half cycle} \\ 0 \text{ to } 1/2}} g \sqrt{2} \cos \pi n x \, dx$$

$$= 2\left(\int_0^{1/4} \sqrt{2} \cos 2\pi n x \, dx - \int_{1/4}^{1/2} \sqrt{2} \cos 2\pi n x \, dx \right)$$

$$= \begin{cases} 0 & n \text{ even} \\ 4 \int_0^{1/4} \sqrt{2} \cos 2\pi n x \, dx & n \text{ odd} \end{cases}$$

$$a_{n\,\text{odd}} = +4 \frac{\sqrt{2}}{2\pi n} \sin 2\pi n x \Big|_0^{1/4}$$

$$= +4 \frac{\sqrt{2}}{2\pi n} \sin 2\pi n x \Big|^{1/4}$$

$$= \frac{+4\sqrt{2}}{2\pi n} \sin \frac{\pi n}{2}$$

$$= \frac{4\sqrt{2}}{2\pi n} (-1)^{\frac{n+1}{2}} \qquad \text{since } \sin \frac{\pi n}{2} = \pm 1$$

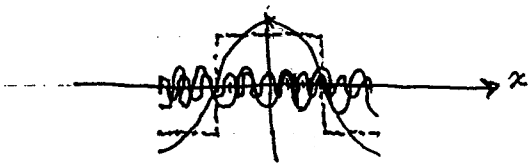
check: $\frac{1}{3} - \frac{1}{5} + \frac{1}{7} - \frac{1}{9} \ldots = \frac{\pi}{4}$

$\therefore \sum \frac{4}{\pi n} (-1)^{\frac{n-1}{2}} = \frac{4}{\pi} \cdot \frac{\pi}{4} = 1$ ✓

(1 = g(0), see g(x) below)

$b_n = 0$ since b_n is even func and sines are odd funcs

$$\therefore \quad g(x) = \sum_{n=0}^{\infty} \frac{*4}{\pi n} (-1)^{\frac{n+1}{2}} \cos 2\pi n x \qquad \text{(see check above)}$$



First term slightly higher than square wave. others start forming corners.