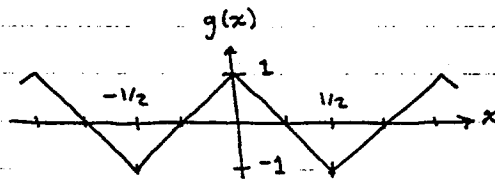


# Fourier Series - Examples - Triangle Wave

ex: Triangle Wave



$$g(x) = \sum_{n=0}^{\infty} (a_n \sqrt{2} \cos 2\pi n x + b_n \sqrt{2} \sin 2\pi n x)$$

$a_0 = \text{ave value} = 0$

$$\begin{aligned} a_{n \neq 0} &= (g, \sqrt{2} \cos 2\pi n x) = \int_{-1/2}^{1/2} g \sqrt{2} \cos 2\pi n x \, dx \\ &= 2 \int_0^{1/2} g \sqrt{2} \cos 2\pi n x \, dx \quad \text{double } \int \text{ by symmetry} \\ &= 2 \int_0^{1/2} (1-2x) \sqrt{2} \cos 2\pi n x \, dx \end{aligned}$$



$$a_{n \neq 0} = \begin{cases} 2 \cdot 2 \int_0^{1/2} (1-2x) \sqrt{2} \cos 2\pi n x \, dx & n \text{ odd} \\ 0 & n \text{ even} \end{cases}$$

$$a_{n \text{ odd}} = 4 \int_0^{1/2} 2x \sqrt{2} (-\cos 2\pi n x) \, dx \quad \text{by horizontal flip}$$

$$= -8\sqrt{2} \int_0^{1/2} x \cos 2\pi n x \, dx$$

$$= \frac{-8\sqrt{2}}{(2\pi n)^2} \int_0^{1/2} 2\pi n x \cos 2\pi n x \, d(2\pi n x)$$

$$= \frac{-8\sqrt{2}}{(2\pi n)^2} \int_0^{\pi n} y \cos y \, dy \quad \text{change of vars}$$

$$= \frac{-8\sqrt{2}}{(2\pi n)^2} (\cos \pi n + \pi n \sin \pi n - \cos 0 + 0 \sin 0)$$

$$a_{n \text{ odd}} = \frac{16\sqrt{2}}{(2\pi n)^2} \quad n \text{ odd}, \quad 0 \quad n \text{ even}$$

$b_n = 0$  since  $g$  is even func and  $\sin$  is odd