DEF: N-dimensional Fourier series for $f(x)$ on domain $\left[-\frac{1}{2},\frac{1}{2}\right]^N$ (or other unit hypercube such as $[0,1]^N$) $\equiv$

$$f(\vec{x}) = \sum_{\vec{n}} \left[a_{\vec{n}} \sqrt{2}\cos(2\pi\vec{n}\circ\vec{x}) + b_{\vec{n}}\sqrt{2}\sin(2\pi\vec{n}\circ\vec{x}) \right]$$

where $\vec{n}$ ranges over all vectors of form $(n_1, \ldots, n_N)$ with integer entries and first nonzero entry restricted to positive values.

NOTE: The restriction that the first nonzero entry in $\vec{n}$ be positive avoids having essentially the same cosine or sine term appear twice in the series owing to the similarity of sinusoids with positive and negative arguments:

$$\cos(x) = \cos(-x)$$

$$\sin(x) = \sin(-x)$$

In other words, the restriction on $\vec{n}$ eliminates $-\vec{n}$ from the series.

NOTE: $\sqrt{2}\cos(2\pi\vec{n}\circ\vec{x}) = \sqrt{2}\cos(2\pi[n_1x_1 + n_2x_2])$ in two dimensions.

NOTE: $\sqrt{2}\cos(2\pi n_1x_1 + n_2x_2)$ [or $\sqrt{2}\sin(2\pi n_1x_1 + n_2x_2)$] resembles water waves with crests lying on parallel lines.

TOOL: $\vec{n}$ is $\perp$ (is perpendicular to) the wave crests of $\sqrt{2}\cos(2\pi\vec{n}\circ\vec{x})$

TOOL: The spacing, d, of wave crests of $\sqrt{2}\cos(2\pi\vec{n}\circ\vec{x})$ is found by solving the following equation:

$$\vec{n}\circ d\frac{\vec{n}}{|\vec{n}|} = 1 \qquad \text{(so argument of cos() = } 2\pi\text{; crests occur where cos() = 1)}$$

or

$$d = \frac{1}{|\vec{n}|}$$