Fourier Series - N-dimensional series - Orthogonal basis functions

verify: N-dimensional Fourier basis functions are orthogonal

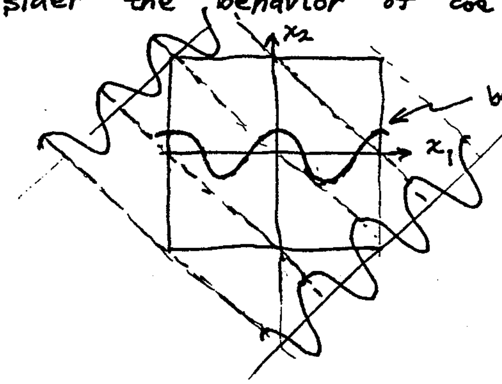
ex: $(\sqrt{2}\cos 2\pi\vec{n}\circ\vec{x}, \sqrt{2}\cos 2\pi\vec{m}\circ\vec{x})$ $\quad \vec{n}\neq\vec{m}$

$$= \int_{-1/2}^{1/2}\cdots\int_{-1/2}^{1/2} 2\cos 2\pi\vec{n}\circ\vec{x}\,\cos 2\pi\vec{m}\circ\vec{x}\; dx_1\ldots dx_N$$

identity: $2\cos A\cos B = \cos(A+B)+\cos(A-B)$

$$(\,,\,) = \int_{-1/2}^{1/2}\cdots\int_{-1/2}^{1/2} \cos[2\pi(\vec{n}+\vec{m})\circ\vec{x}] + \cos[2\pi(\vec{n}-\vec{m})\circ\vec{x}]\; dx_1\ldots dx_N$$

Consider the behavior of $\cos 2\pi(\vec{n}+\vec{m})\circ\vec{x}$ in x_1 direction:



The cross section looks like $\cos(2\pi(n_1+m_1)x_1)$.
When we integrate $\int_{-1/2}^{1/2}\cos 2\pi(\vec{n}+\vec{m})\circ\vec{x}\,dx_1$ we

are integrating an integral number of cycles of a cosine regardless of the value of $x_2,\ldots,x_N$.
Thus, we get zero unless $n_1+m_1=0$.

A similar argument applies to other dimensions, and we discover that we get a zero integral unless $\vec{n}+\vec{m}=(0,\ldots,0)$. This cannot happen since $n_1>0$ and $m_1>0$. Similarly, our integral for $\cos[2\pi(\vec{n}-\vec{m})\circ x]$ is zero unless $\vec{n}-\vec{m}=0$.

But $\vec{n}\neq\vec{m}$ was assumed. $\therefore \int = 0$

Thus, the basis functions are orthogonal.