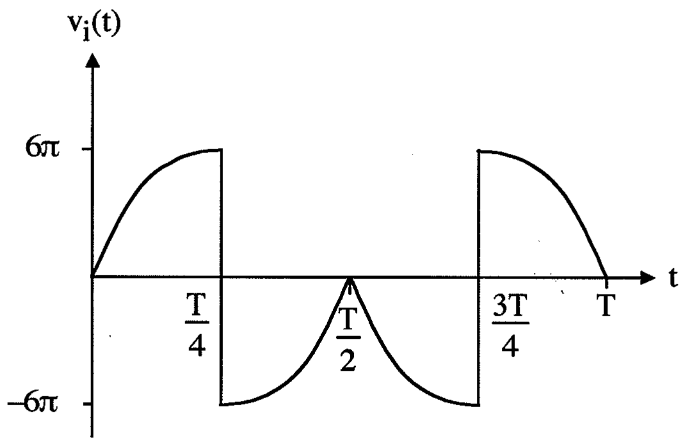
Ex:



T = one period of $v_i(t) = 2\pi/60$ s

$$v_i(t) = \begin{cases} 6\pi \sin(\omega_0 t)\,\text{V} & 0 < t \le T/4 \\ -6\pi \sin(\omega_0 t)\,\text{V} & T/4 < t \le T/2 \\ 6\pi \sin(\omega_0 t)\,\text{V} & T/2 < t \le 3T/4 \\ -6\pi \sin(\omega_0 t)\,\text{V} & 3T/4 < t \le T \end{cases}$$

Find numerical values of coefficients a_v, a_1, a_2, b_1, and b_2 for the Fourier series for $v_i(t)$:

$$v_i(t) = a_v + \sum_{k=1}^{\infty} a_k \cos(k\omega_0 t) + b_k \sin(k\omega_0 t)$$

Sol'n: Because $v_i(t)$ has equal area above and below 0V, the average value of $v_i(t)$ is zero:

$$a_v = 0\,\text{V}$$

Since $v_i(t)$ is periodic, it consists of copies of the above waveform repeating to the left and right.

If we reflect $v_i(t)$ about the vertical axis, we obtain $v_i(t)$. Thus, $v_i(t) = v_i(-t)$ and $v_i(t)$ is an even function.

$\therefore\ b_k = 0\text{V}$ for all k

$b_1 = 0\text{V}$ and $b_2 = 0\text{V}$

If we shift $v_i(t)$ right (or left) by one-half period and then flip it upside down, we obtain $v_i(t)$. Thus, $v_i(t)$ has shift flip symmetry and even numbered Fourier coefficients are zero.

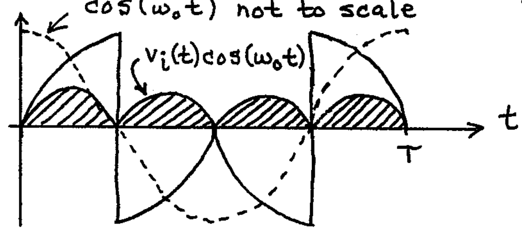
$\therefore\ a_k = 0\text{V}$ for all even k

$a_2 = 0\text{V}$

That leaves us with the calculation of a_1:

$$a_1 = \frac{2}{T}\int_0^T v_i(t)\cos(1\cdot\omega_0 t)\,dt$$

A sketch of $v_i(t)$, $\cos(\omega_0 t)$, and $v_i(t)\cos(\omega_0 t)$ reveals how the calculation of a_1 reduces to the computation of a single integral.



From the sketch, it is apparent that four times the integral from 0 to $T/4$ of $v_i(t)\cos(\omega_o t)$, (i.e., the hatched area), gives the value of a_1:

$$a_1 = 4 \cdot \frac{2}{T} \int_0^{T/4} v_i(t) \cos(\omega_o t)\, dt \ \text{ V}$$

$$" = \frac{8}{T} \int_0^{T/4} 6\pi \sin(\omega_o t) \cos(\omega_o t)\, dt \ \text{ V}$$

$$" = \frac{8}{T} \cdot 6\pi \int_0^{T/4} \frac{\sin(2\omega_o t)}{2}\, dt \ \text{ V}$$

$$a_1 = \frac{24\pi}{T} \int_0^{T/4} \sin(2\omega_o t)\, dt \ \text{ V}$$

The fundamental frequency, ω_o, is given by

$$\omega_o = \frac{2\pi}{T}.$$

We may choose any convenient value for T without changing the value of Fourier coefficients.

For convenience, we might set $T = 2\pi$, for example. This simplifies the calculation. On the other hand, retaining T as a symbolic variable allows us to verify that all the factors of T cancel out. This is a convenient consistency check.

$$a_1 = \frac{24\pi}{T} \cdot \left. \frac{-\cos(2\omega_0 t)}{2\omega_0} \right|_0^{T/4} V$$

$$= \frac{24\pi}{T} \cdot \frac{-\cos\left(2 \cdot \frac{2\pi}{T} \cdot \frac{T}{4}\right) - -\cos(0)}{2 \cdot \frac{2\pi}{T}} V$$

$$= 6\,[1 - \cos(\pi)]\,V$$

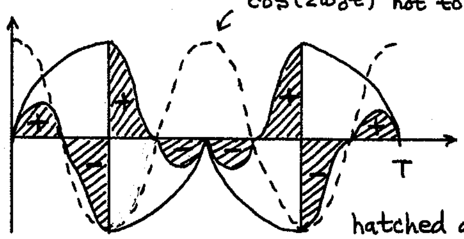
$$= 6\,[1 - -1]\,V$$

$$a_1 = 12V$$

Note: Sketches illustrate why a_2, b_1, and b_2 are zero.

$\cos(2\omega_0 t)$ not to scale

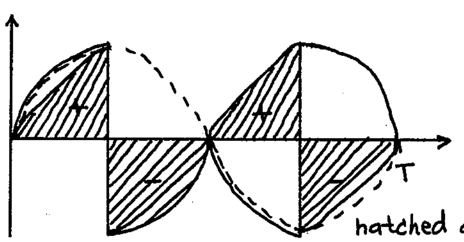
a_2:



hatched area = $\int v_i \cos(2\omega_0 t)$

[+ and − areas sum to 0] " " $= 0$

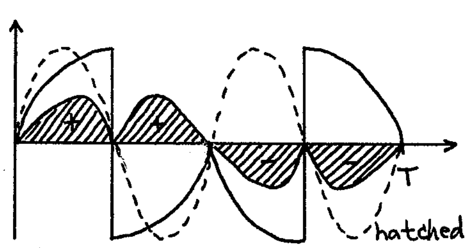
b_1:



hatched area = $\int v_i \sin(\omega_0 t)$

$= 0$

b_2:



hatched area = $\int v_i \sin(2\omega_0 t)$

$= 0$