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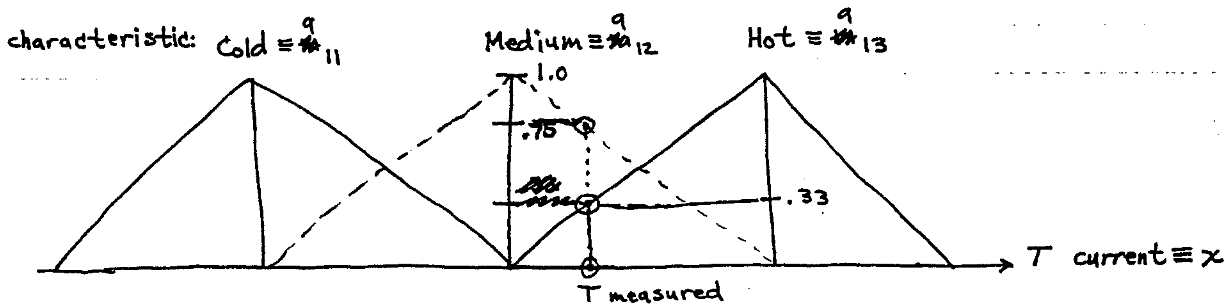
3 / 1994

~~tool~~: A fuzzy logic membership function specifies how much of a particular characteristic an input value posseses.

ex: We want to control gas flow to a furnace as a function of the current temperature inside the furnace and the desired temperature.

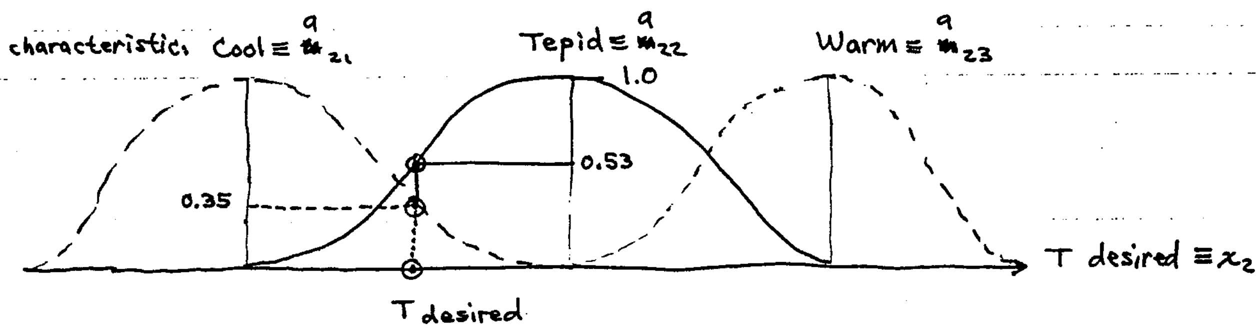
We create membership functions for current temperature and desired temperature.



observe: we have 3 triangular membership functions. For the measured temperature, T, we have:

0.0	membership	in	Cold	$= a_{11}$ (T current)
0.33	"	"	Medium	$= a_{12}$ (T current)
0.75	"	"	Hot	$= a_{13}$ (T current)

We use a different shape for the membership function for T desired. (We choose the shape just to illustrate the range of shape choices available.)



We have:

0.35	membership	in	Cool	$= a_{21}$ (T desired)
0.53	"	"	Tepid	$= a_{22}$ (T desired)
0.0	"	"	Warm	$= a_{23}$ (T desired)

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observe: 1) The membership function has a maximum value of 1.

2) The membership function M_i has a value of zero (0) when the value of input, x_i, lies under the maximum point for a neighboring characteristic.

tool: We compute the combined membership for different types of characteristics by computing the fuzzy AND of membership functions for each characteristic.

notn: $\tilde{\wedge}$ ≡ fuzzy AND

$\circ$ ≡ dot product

tool: The fuzzy AND of a and b is the minimum of a and b:

$$a \,\tilde{\wedge}\, b = \min(a, b)$$

ex: $.6 \,\tilde{\wedge}\, .4 = \min(.6, .4) = .4$

ex: For our furnace control example we have

$$\begin{aligned} m_{11} &= \text{membership function for } (T \text{ current is Cold}, T \text{ desired is Cool}) \\ &= a_{11}(T\text{ current}) \,\tilde{\wedge}\, a_{21}(T\text{ desired}) \\ &= \min(a_{11}(T\text{ current}), a_{21}(T\text{ desired})) \\ &= \min(0.0, 0.35) \\ &= 0.0 \end{aligned}$$

$$\begin{aligned} m_{32} &= \text{membership function for } (T \text{ current is Hot}, T \text{ desired is Tepid}) \\ &= a_{13}(T\text{ current}) \,\tilde{\wedge}\, a_{22}(T\text{ desired}) \\ &= \min(a_{13}(T\text{ current}), a_{22}(T\text{ desired})) \\ &= \min(0.75, 0.53) \\ &= 0.53 \end{aligned}$$

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ex: (cont) The complete matrix of pairwise memberships is as follows:

$$\begin{bmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \end{bmatrix} = \begin{bmatrix} 0.0 & 0.0 & 0.0 \\ 0.33 & 0.33 & 0.0 \\ 0.35 & 0.53 & 0.0 \end{bmatrix}$$

tool: If the membership for either characteristic is zero, then a row or column in the pairwise membership matrix will be zero.

note: We can have more than 2 characteristics. If we have 3 characteristics with membership functions, a_{1i}, a_{2j}, a_{3k} $i=1,\ldots,N$ $j=1,\ldots,M$ $k=1,\ldots,P$, then we compute the triplet memberships using fuzzy AND's: $a_{1i} \tilde{\wedge} a_{2j} \tilde{\wedge} a_{3k} = m_{ijk} = \min(a_{1i}, a_{2j}, a_{3k})$.

In other words, we generalize to 3 or more characteristics (one for each input variable) in the obvious way.

tool: If the membership functions are triangular, then the pairwise membership functions are pyramids.

ex: