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def: membership function of $\tilde{C} = \tilde{A}$ interset $\tilde{B} \equiv$

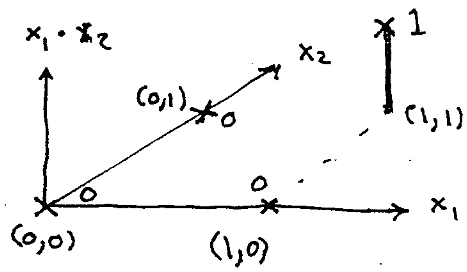
$$m_{\tilde{C}}(x) = \min\left(m_{\tilde{A}}(x), m_{\tilde{B}}(x)\right)$$

notn: the following are equivalent

$$\tilde{C} = \tilde{A} \text{ intersect } \tilde{B}$$
$$\tilde{C} = \tilde{A} \cap \tilde{B}$$
$$\tilde{C} = \tilde{A} \text{ fuzzy AND } \tilde{B}$$
$$\tilde{C} = \tilde{A} \mathbin{\tilde{\wedge}} \tilde{B}$$
$$\tilde{C} = \tilde{A} \cdot \tilde{B}$$
$$\tilde{C} = \tilde{A}\tilde{B}$$

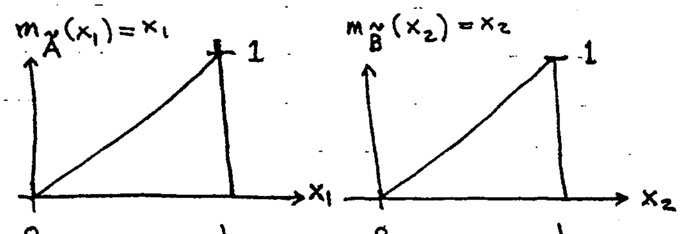
ex: The fuzzy AND extends the notion of AND found in Boolean algebra. We show this pictorially.

digital AND:
$x_1 = 0$ or 1
$x_2 = 0$ or 1
$x_1 \cdot x_2 = 0$ or 1

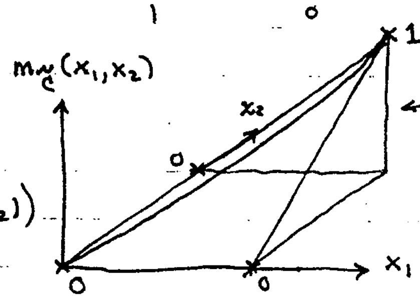


x_1	x_2	$x_1 \cdot x_2$
0	0	0
1	0	0
0	1	0
1	1	1

fuzzy AND:
$x_1 \in [0,1]$
$x_2 \in [0,1]$
$m_{\tilde{A}}(x_1) \in [0,1]$
$m_{\tilde{B}}(x_2) \in [0,1]$
$m_{\tilde{C}}(x_1, x_2) \in [0,1]$



$$m_{\tilde{C}}(x_1, x_2) = \min\left(m_{\tilde{A}}(x_1), m_{\tilde{B}}(x_2)\right)$$
$$= \min(x_1, x_2)$$



Looks like one corner of a square pyramid.

We reproduce the digital AND results, but the fuzzy also defines an output for $0 < x_1 < 1$ and $0 < x_2 < 1$.