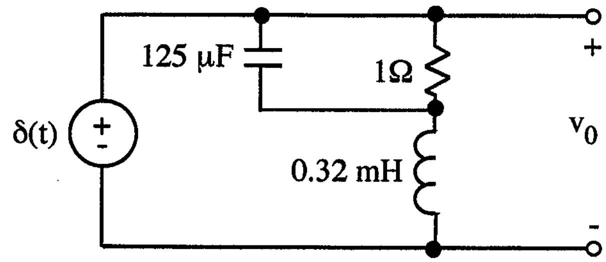
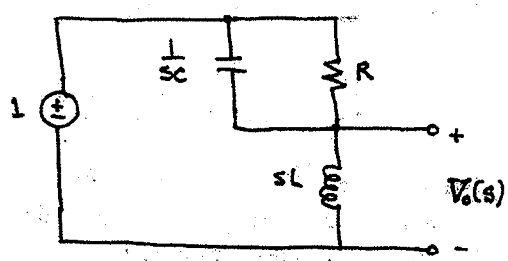
3.10.1 For the circuit illustrated below, write a numerical time-domain expression for $v_0(t)$ where $t > 0$. There is no energy stored in the circuit initially.



Answer: $v_0(t) = \left[\delta(t) - \frac{25}{3}\text{k}\, e^{-4\text{k}t}\sin(3\text{k}t)\right]u(t)$ V

Solution:

s-domain model: $\mathcal{L}\{\delta(t)\} = 1$



V-divider: $V_0(s) = \dfrac{sL}{sL + R\|\frac{1}{sC}}$ $\qquad R\|\frac{1}{sC} = \dfrac{R/sC}{R + 1/sC} = \dfrac{R}{1+sRC}$

$= \dfrac{sL}{sL + \frac{R}{1+sRC}} = \dfrac{sL(1+sRC)}{sL(1+sRC)+R}$

$= \dfrac{sL + s^2LRC}{sL + s^2LRC + R} = \dfrac{s^2 + \frac{1}{RC}s}{s^2 + \frac{1}{RC}s + \frac{1}{LC}}$

$= 1 - \dfrac{1/LC}{\underbrace{s^2 + \frac{1}{RC}s + \frac{1}{LC}}_{(s+a)^2+\omega^2 \,=\, s^2+2as+a^2+\omega^2}}$

Now, $a = \frac{1}{2RC}$ $\qquad \omega^2 = \underbrace{\frac{1}{LC} - \left(\frac{1}{2RC}\right)^2}_{9M}$

$a = 4k$ $\qquad \omega = 3k$

$\frac{1}{RC} = \dfrac{1}{1\cdot\frac{1mF}{8}} = 8k$

$\frac{1}{LC} = \dfrac{1}{0.32m\cdot\frac{1mF}{8}} = \frac{1}{0.04}M$

or $\frac{1}{LC} = 25M$

$\left(\frac{1}{2RC}\right) = (4k)^2 = 16M$

Now we use $\mathcal{L}\left\{\frac{\omega}{(s+a)^2+\omega^2}\right\} = e^{-at}\sin(\omega t)$. We write

$$V_o(s) = 1 - \frac{1/LC}{\omega}\frac{\omega}{(s+a)^2+\omega^2} = 1 - \frac{25M}{3K}\frac{3K}{(s+4K)^2+(3K)^2}$$

$$\therefore\ v_o(t) = \left[\delta(t) - \frac{25}{3}K\, e^{-4Kt}\sin(3Kt)\right]u(t) \quad V$$

Note: $\mathcal{L}^{-1}\{1\} = \delta(t)$.