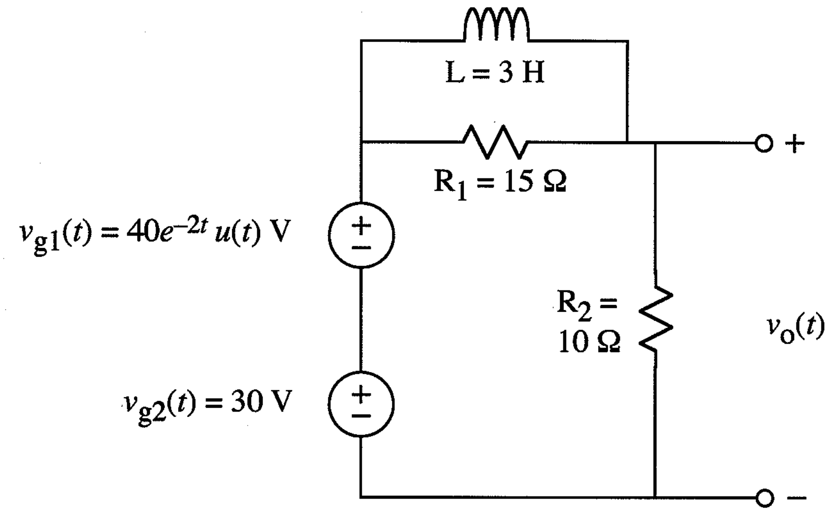
**Ex:**




a) Write the Laplace transform, $V_{g1}(s)$, of $v_{g1}(t)$.

b) Draw the s-domain equivalent circuit, including sources $V_{g1}(s)$ and $V_{g2}(s)$, components, initial conditions for L, and terminals for $V_o(s)$. Note that the 30 V source is on for all time.

c) Write an expression for $V_o(s)$. You may write parallel impedances using the || operator without having to simplify them.

d) Apply the initial value theorem to find $\lim\limits_{t\to 0^+} v_o(t)$.

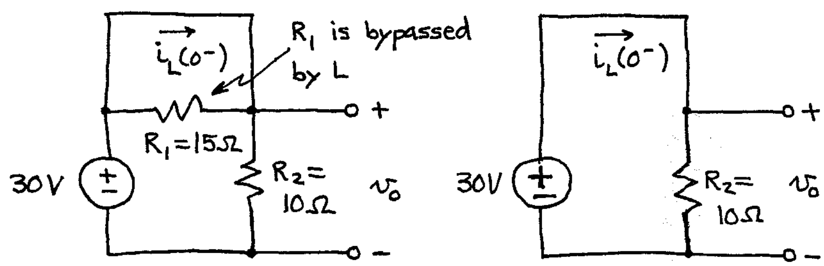
Sol'n: a) From a table of Laplace transforms we have

$$\mathcal{L}\{v_{g1}(t)\} = \mathcal{L}\{40e^{-2t}u(t)V\} = \frac{40}{s+2}\ V$$

Note: we append units of V to our answer, although our expression has units V·sec owing to integration over time when we take the Laplace transform. s has units of 1/sec and the 2 in the denominator has units of 1/sec in $e^{-2t}$. Thus, our answer should technically be $\frac{40V}{s+2/\text{sec}}$.

b) We first find the initial conditions for the L. At $t=0^-$, the L acts like a wire, and only the $v_{g2}$ source has a nonzero value.

$t=0^-$ circuit model:



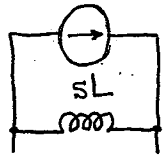

Our circuits simplifies

$$i_L(0^-) = \frac{30V}{R_2} = \frac{30V}{10\Omega}$$

$$i_L(0^-) = 3A$$

Although we may model the initial conditions on L as a series voltage source or a parallel current source, a parallel current source is convenient owing to the R in parallel with L.

$i_L(0^-)/s = 3/s$ A

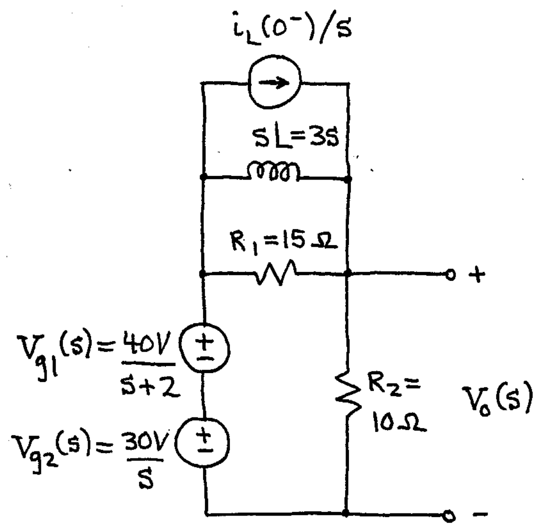



From part (a) we have $\mathcal{L}\{v_{g1}(t)\} = \dfrac{40\,V}{s+2}$.

For $v_{g2}(t)$, we have $\mathcal{L}\{v_{g2}(t)\} = \mathcal{L}\{30V\}$.

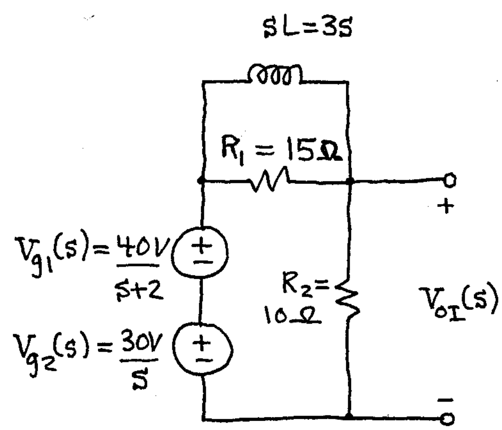
We treat $30V$ as $30V\,u(t)$. Thus, $\mathcal{L}\{v_{g2}(t)\} = \dfrac{30V}{s}$.

$s$-domain model:

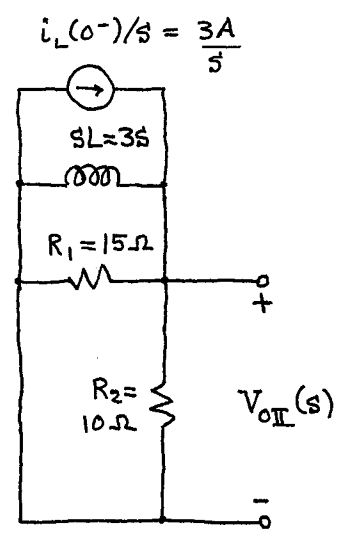



c) We may use superposition to find $V_o(s)$.
We have two circuits:

circuit I:




circuit II:




circuit I is V-divider    circuit II is solved by Ohm's law

We write down $V_{oI}(s)$ and $V_{oII}(s)$ by inspection:

$$V_{oI}(s) = \left(\frac{40V}{s+2} + \frac{30V}{s}\right) \frac{R_2}{R_2 + R_1 \| sL}$$

$$V_{oII}(s) = \frac{i_L(0^-)}{s} \cdot R_2 \| R_1 \| sL$$

$$V_o(s) = V_{oI}(s) + V_{oII}(s) \qquad \text{(we sum the } V\text{'s.)}$$

or $$V_o(s) = \left(\frac{40V}{s+2} + \frac{30V}{s}\right) \frac{R_2}{R_2 + R_1 \| sL} + \frac{i_L(0^-)}{s} R_2 \| R_1 \| sL$$

We may simplify the expression to a ratio of polynomials. We begin by finding an expression for $R \| sL$.

We write $R \| sL$ as $sLR \cdot \frac{1}{sL} \| \frac{1}{R} = \frac{sLR}{sL+R} = \frac{sR}{s+R/L}$

Note: we are using the identity $\frac{1}{a} \| \frac{1}{b} = \frac{1}{a+b}$.

Thus, $R_2 \| R_1 \| sL = \dfrac{sL \cdot R_1 \| R_2}{sL + R_1 \| R_2}$ where $R_1 \| R_2 = 10\Omega \| 15\Omega$

$$= 5\Omega \cdot 2 \| 3$$

$$= 5 \cdot \frac{6\Omega}{5} = 6\Omega$$

$$= \frac{s \, R_1 \| R_2}{s + \frac{R_1 \| R_2}{L}}$$

$$= \frac{s \cdot 6\Omega}{s + \frac{6\Omega}{3H}}$$

$$R_2 \| R_1 \| sL = \frac{s \cdot 6\Omega}{s+2}$$

Now we substitute $R_1 \| sL = \dfrac{sR_1}{s + R_1/L}$ in the 1st term.

$$\frac{R_2}{R_2 + R_1 \| sL} = \frac{R_2}{R_2 + \frac{sR_1}{s+R_1/L}} = \frac{R_2(s+R_1/L)}{R_2(s+R_1/L)+sR_1}$$

$$= \frac{R_2}{R_1+R_2} \frac{s+R_1/L}{s+\frac{R_1\|R_2}{L}}$$

$$= \frac{10\ \Omega}{10+15\ \Omega} \frac{s+15\Omega/3\text{H}}{s+6\Omega/3\text{H}}$$

$$\frac{R_2}{R_2+R_1\|sL} = \frac{2}{5}\frac{s+5}{s+2}$$

Plugging in values, we find $V_o(s)$:

$$V_o(s) = \left(\frac{40\text{V}}{s+2} + \frac{30\text{V}}{s}\right)\frac{2}{5}\frac{s+5}{s+2} + \frac{3\text{A}\cdot 6\Omega\cdot s}{s}\frac{}{s+2}$$

$$V_o(s) = \frac{[40\text{V}s + 30(s+2)](2/5)(s+5)+18\text{V}s(s+2)}{s(s+2)^2}$$

$$= \frac{28s^2+84s+120+18s^2+36s}{s(s+2)^2}\ \text{V}$$

$$V_o(s) = \frac{46s^2+120s+120}{s(s+2)^2}\ \text{V}$$

d) By the initial value theorem $\lim_{t\to 0^+} v_o(t) = \lim_{s\to\infty} sV_o(s)$.

We only use the highest power of $s$ in the numurator and denominator as these dominate as $s \to \infty$.

$$\lim_{t\to 0^+} v_o(t) = \lim_{s\to\infty} \frac{\cancel{s}\cdot 46\cancel{s^2}\,V}{\cancel{s}\cdot\cancel{s^2}} = 46\ V$$

Note: We may also apply the initial value theorem to the unsimplified $V_o(s)$.

We observe that $\lim_{s\to\infty} R \| sL = R \| \infty = R$.

$$\lim_{s\to\infty} sV_o(s) = \lim_{s\to\infty} \cancel{s}\left(\frac{40V}{s+2} + \frac{30V}{\cancel{s}}\right)\frac{R_2}{R_1+R_2} + \frac{\cancel{s}i_L(0^-)}{\cancel{s}} R_2 \| R_1$$

ignore 2 since $2 \ll s \to \infty$

$$\| \quad = 70V\cdot\frac{R_2}{R_1+R_2} + i_L(0^-)\,R_2 \| R_1$$

$$\| \quad = 70V\cdot\frac{10\Omega}{10+15\Omega} + 3A\cdot 6\Omega$$

$$\| \quad = 28V + 18V$$

$$\lim_{t\to 0^+} v_o(t) = 46V \quad \checkmark$$