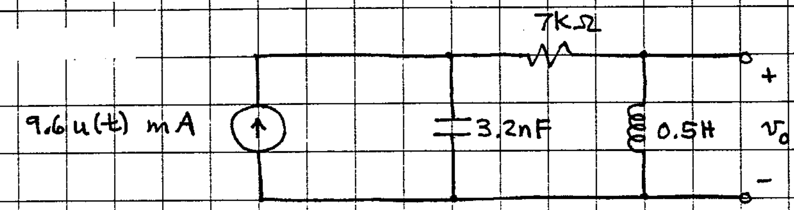


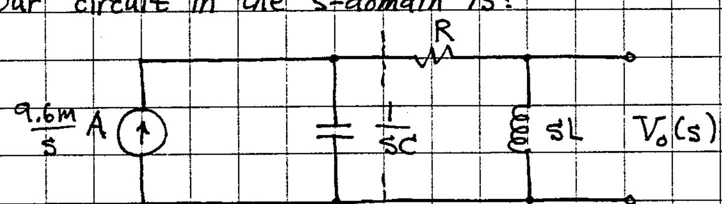
a) Find $V_o(s)$ b) Find $v_o(t=0^+)$ and $v_o(t\to\infty)$ c) Find $v_o(t)$

ans: a) $V_o(s) = \dfrac{3M}{s^2 + 14ks + 625M}$ b) $v_o(t=0^+) = 0\text{ V}$, $v_o(t\to\infty) = 0\text{ V}$ c) $v_o(t) = 125e^{-7kt} \cdot \sin 24kt \cdot u(t)$

soln: We assume zero initial conditions, (since none were specified).

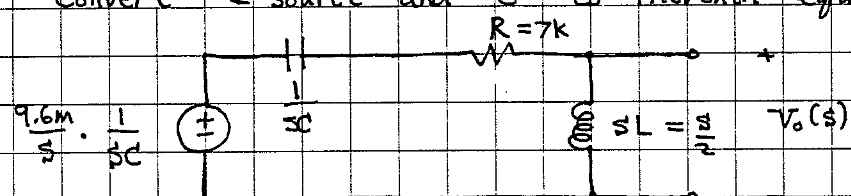
$$\mathcal{L}\{9.6\,u(t)\text{ mA}\} = \frac{9.6m}{s}\text{ A}$$

∴ our circuit in the s-domain is:



← take Thev equiv

Convert I-source and C to Thevenin equiv.



$$V_o(s) = \frac{sL}{sL + \frac{1}{sC} + R} \cdot \frac{9.6m}{s^2C} \quad \text{V-divider}$$

$$\text{or } V_o(s) = \frac{\cancel{s^2}}{\left(s^2 + \frac{1}{LC} + s\frac{R}{L}\right)} \frac{9.6m/C}{\cancel{s^2}}$$

[mult top & bottom by $\frac{s}{L}$ to get polynomials and coeff (of highest order term in denominator) = 1]

$$\text{or } V_o(s) = \frac{(9.6m/C = 3M)}{s^2 + 14ks + 625M}.$$

b) Initial value theorem: $v_o(t=0^+) = \lim_{s\to\infty} s V_o(s)$

Final value theorem: $v_o(t\to\infty) = \lim_{s\to 0} s V_o(s)$

$$s V_o(s) = \frac{s \cdot 3M}{s^2 + 14ks + 625M}$$

$$v_o(t=0^+) = \lim_{s\to\infty} \frac{s \cdot 3M}{s^2 + 14ks + 625M}$$

$$= \lim_{s\to\infty} \frac{s \cdot 3M}{s^2}$$

only highest order term top and bottom matter (because they dominate)

$$= \lim_{s\to\infty} \frac{3M}{s}$$

$$= \frac{3M}{\infty} = 0 \text{ V}$$

$$v_o(t\to\infty) = \lim_{s\to 0} \frac{s \cdot 3M}{s^2 + 14ks + 625M}$$

$$= \lim_{s\to 0} s \cdot \underbrace{\lim_{s\to 0} \frac{3M}{s^2 + 14ks + 625M}}$$

$$= 0 \cdot \frac{3M}{625M}$$

$$= 0 \text{ V}$$

can factor out this constant term $= \frac{3M}{625M}$; i.e. $\lim k f(x) = k \lim f(x)$.

Comment: The virtue of these theorems is that they tell us about initial conditions and stability (final value) without requiring us to compute an inverse Laplace transform.

We can also use these theorems to check our inverse Laplace transforms.

c) We could use partial fractions, in theory, but our poles are a complex conjugate pair:

$$s^2 + 14ks + 625M = 0 \quad \Rightarrow \quad s = -\frac{14k}{2} \pm \sqrt{\left(\frac{14k}{2}\right)^2 - 625M}$$

$$s = -7k \pm j\,24k$$

Note: $7^2 = 25^2 - 24^2 = 25 + 24$
$3^2 = 5^2 - 4^2 = 5 + 4$
$5^2 = 13^2 - 12^2 = 13 + 12$
In general, $(n+1)^2 - n^2 = 2n+1 = (n+1) + n$.
$(n+1)+n$ is always odd. If $(n+1)+n = \text{odd}^2$ then we have Pythagorean triple.

Note: The roots are $-a_1$ and $-a_2$, but our factored polynomial is $(s+a_1)(s+a_2)$. This <u>is</u> correct.

For a pair of complex conjugate poles, we follow a simpler procedure:

Observe that complex conjugate poles correspond to damped sinusoids, (always).

$$\mathcal{L}\{k_1 e^{-at}\sin(\omega t)u(t)\} = \frac{k_1\omega}{(s+a)^2 + \omega^2} = \frac{k_1\omega}{s^2 + 2as + a^2 + \omega^2}$$

$$\mathcal{L}\{k_2 e^{-at}\cos(\omega t)u(t)\} = \frac{k_2(s+a)}{(s+a)^2 + \omega^2} \quad \frac{k_2(s+a)}{s^2 + 2as + a^2 + \omega^2}$$

In the general case, we will have a term of form:

$$F(s) = \frac{bs + d}{s^2 + e\cdot s + f} \qquad b, d, e, f \text{ constants}$$

We identify $e = 2a \Rightarrow \boxed{a = \frac{e}{2}}$

$f = a^2 + \omega^2 \Rightarrow \boxed{\omega = \sqrt{f - a^2} = \sqrt{f - \left(\frac{e}{2}\right)^2}}$

For the numerator, we have: $k_1\omega + k_2(s+a) = bs+d$.

$\therefore \boxed{k_2 = b}$ and $k_1\omega + k_2 a = d$

or $k_1 = \dfrac{d - k_2 a}{\omega} = \dfrac{d - b \cdot a}{\omega}$

or $\boxed{k_1 = \dfrac{d - b\cdot\left(\frac{e}{2}\right)}{\sqrt{f - \left(\frac{e}{2}\right)^2}}}$

$\therefore$ We can write any term for conjugate poles as a sum of a damped sine and cosine.

Here, we have $b = 0$, $d = 3M$, $e = 14k$, $f = 625M$.

$\therefore a = 7k \quad \omega = \sqrt{625M - (7k)^2} = 24k$

Note: $a = -\text{Re}[\text{roots of denom}] = -\text{Re}[\text{poles}]$
$\omega = \text{Im}[\text{roots of denom}] = \text{Im}[\text{poles}]$

$k_2 = 0 \qquad k_1 = \dfrac{3M - 0\cdot 7k}{\omega = 24k} = 125$

$\therefore v_o(t) = k_1 e^{-at}\sin(\omega t)\,u(t) = 125e^{-7kt}\sin(24kt)\,u(t)$.

Note: The major drawback to this approach is that it may be difficult to create a term of form $\dfrac{bs+d}{s^2+es+f}$. In that case we use partial fractions with complex numbers.

If we have the partial fraction expansion terms

$$\frac{K_1}{s+a+j\omega} + \frac{K_1^*}{s+a-j\omega} \qquad K_1 = F(s)(s+a+j\omega)\Big|_{s=-a+j\omega}$$

or $K_1 = F(s=-a+j\omega)\,j2\omega$

then we can use a common denominator:

$$\frac{K_1(s+a-j\omega) + K_1^*(s+a+j\omega)}{s^2+2as+a^2+\omega^2} = \frac{bs+d}{(s+a)^2+\omega^2}$$

$$\text{with } b = K_1 + K_1^* = 2\,\mathrm{Re}[K_1] \qquad \text{from } c+c^* = 2\,\mathrm{Re}[c]$$

$$d = K_1(a-j\omega) + K_1^*(a+j\omega) = 2\,\mathrm{Re}[K_1(a-j\omega)]$$

Here, we have $K_1 = \dfrac{3M\,(s+7k+j24k)}{s^2+14ks+625M}\Bigg|_{s=-7k+j24k}$

$$= \frac{3M}{s+7k+j24k}\Bigg|_{s=-7k+j24k}$$

$$= \frac{3M}{j48k} = -j\frac{3M}{48k}$$

Thus, $b = 2\,\mathrm{Re}\left[-j\dfrac{3M}{48k}\right] = 0$ ✓

$$d = 2\,\mathrm{Re}\left[-j\frac{3M}{48k}\cdot(7k-j24k)\right]$$

$$= 2\,\mathrm{Re}\left[-j\frac{3M\cdot 7k}{48k} - \frac{3M\cdot 24k}{48k}\right]$$

$$= -2\cdot\frac{3M\cdot 24k}{48k} = 3M$$ ✓