EX:

Circuit: current source $i_g(t) = 3e^{-10t}\cos(3t)$ A in parallel with $R = 1\ \Omega$ and $C = 0.1$ F, switch at $t = 0$, output $v_0(t)$ across $+$ and $-$ terminals.
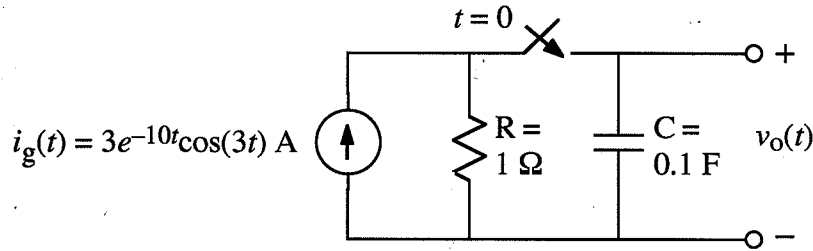
Note: The initial voltage on the capacitor is $v_C(t = 0^-) = 3$ V measured with + on top.

a)   Write the Laplace transform $I_g(s)$ of $i_g(t)$.

b)   Write the Laplace transform $V_0(s)$ of $v_0(t)$. Be sure to include the effects of the initial voltage on the C.

c)   Write a numerical time-domain expression for $v_0(t)$ where $t > 0$. Hint: plug in numbers and look for terms that cancel before computing partial fractions.
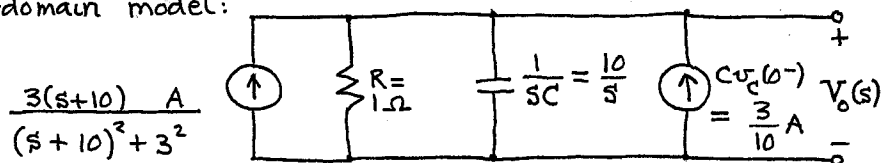
Sol'n: a) From a table of Laplace transforms:

$$\mathcal{L}\{i_g(t)\} = \mathcal{L}\{3e^{-10t}\cos(3t)\ A\} = \frac{3(s+10)}{(s+10)^2 + 3^2}\ A$$

b) It is convenient to use a parallel current source for initial conditions on the Capacitor.

↗ switch acts like $u(t)$

s-domain model:

$\dfrac{3(s+10)}{(s+10)^2 + 3^2}$ A current source in parallel with $R = 1\ \Omega$, $\dfrac{1}{sC} = \dfrac{10}{s}$, and current source $Cv_C(0^-) = \dfrac{3}{10}$ A, output $V_0(s)$.

The output voltage is the total current times the total parallel resistance.

$$V_o(s) = \left(\frac{3(s+10)\,A}{(s+10)^2 + 3^2} + \frac{3}{10}A \right) R \parallel \frac{1}{sC}$$

Simplifying further, we have

$$R \parallel \frac{1}{sC} = R \cdot \left(\frac{1}{1} \parallel \frac{1}{sRC} \right) = \frac{R}{1+sRC} = \frac{1\,\Omega}{1 + \frac{s}{10}} = \frac{10\,\Omega}{s+10}$$

$$\therefore V_o(s) = \left(\frac{3(s+10)\,A}{(s+10)^2 + 3^2} + \frac{3}{10}A \right) \frac{10\,\Omega}{s+10}$$

$$\text{"} = \frac{30V}{(s+10)^2 + 3^2} + \frac{3V}{s+10}$$

$$V_o(s) = \frac{10V \cdot 3}{(s+10)^2 + 3^2} + 3V \cdot \frac{1}{s+10}$$

c) The above form for $V_o(s)$ is

$$V_o(s) = \frac{10V \cdot \omega}{(s+a)^2 + \omega^2} + 3V \cdot \frac{1}{s+a} \quad \text{where } a = 10$$
$$\omega = 3$$

Using a table, we inverse transform $V_o(s)$ by inspection:

$$v_o(t) = \left[10V\, e^{-10t} \sin(3t) + 3V e^{-10t} \right] u(t)$$