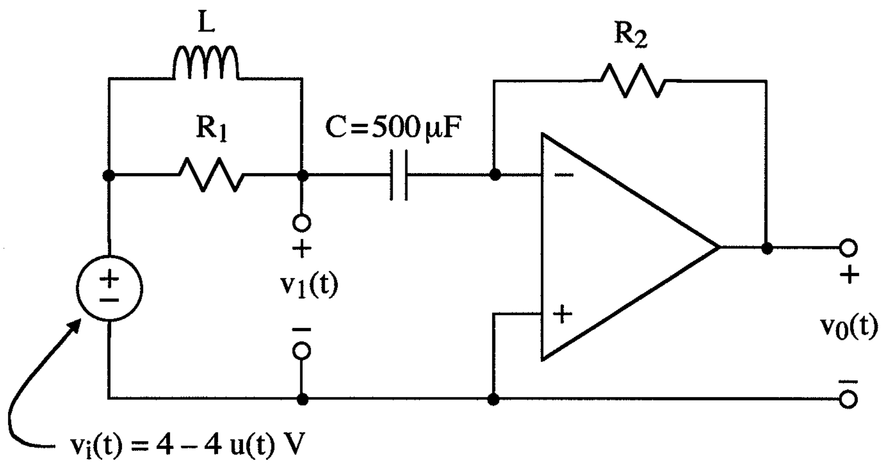
Ex:



The Laplace transform of $v_0(t)$ for the above circuit is as follows:

$$\mathbf{V}_0(s) = 4\text{V} \cdot \frac{R_2}{R_1} \cdot \frac{s + \frac{R_1}{L}}{s^2 + \frac{1}{R_1 C} s + \frac{1}{LC}}$$

Choose numerical values for R_1 and L to make

$$v_1(t) = v_m e^{-\alpha t} \cos(\beta t + \varphi)$$

where $\alpha = \beta = 100$ rad/s.

sol'n: We first observe that $\cos(\beta t + \varphi)$ is the same as a cosine of frequency β plus a sine of frequency β:

$$\cos(\beta t + \varphi) = \cos(\varphi)\cos(\beta t) - \sin(\varphi)\sin(\beta t)$$

Thus, we may rewrite $v_1(t)$ as follows:

$$v_1(t) = v_m \cos(\varphi) e^{-\alpha t} \cos(\beta t) - v_m \sin(\varphi) e^{-\alpha t} \sin(\beta t)$$

Taking the Laplace transform, we have

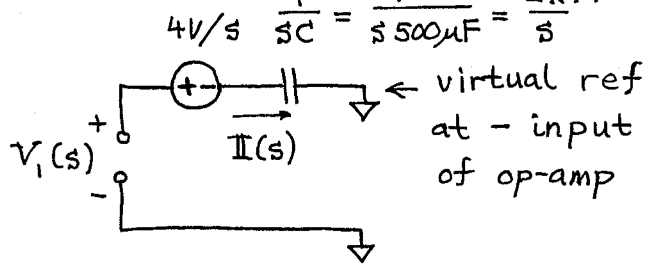
$$\mathcal{L}\{v_1(t)\} = V_1(s) = v_m \cos(\varphi) \frac{s+\alpha}{(s+\alpha)^2 + \beta^2} - v_m \sin(\varphi) \frac{\beta}{(s+\alpha)^2 + \beta^2}$$

From the circuit diagram, we see that $V_1(s)$ is the s-domain voltage across C.

The initial condition on C, found using $v_C(0^+) = v_C(0^-)$, is $v_C(t = 0^-) = 4V$ (from $v_i(t < 0) = 4V$).

This yields the following circuit for $V_i(s)$ in which $I(s)$ remains to be determined:

$$\frac{1}{sC} = \frac{1}{s\,500\mu F} = \frac{2k/F}{s}$$



From the circuit diagram in the problem statement, we have current $I(s)$ determining $V_o(s)$:

$$V_o(s) = -I(s)R_2$$

or $$I(s) = \frac{-V_o(s)}{R_2}$$

Using the expression for $V_o(s)$ given in the problem statement, we have the following expression for $I(s)$:

$$I(s) = -\frac{4V}{R_1}\frac{s + R_1/L}{s^2 + \frac{1}{R_1C}s + \frac{1}{LC}}$$

Now we can write the following eq'n:

$$V_1(s) = \frac{4V}{s} + I(s)\cdot\frac{1}{sC}$$

$$= \frac{4V}{s} + \frac{-4V}{R_1}\,\frac{s + R_1/L}{s^2 + \frac{1}{R_1C}s + \frac{1}{LC}}\cdot\frac{1}{sC}$$

$$= \frac{4V\cdot R_1C(s^2 + s/R_1C + 1/LC) - 4V(s + R_1/L)}{s\cdot R_1C(s^2 + s/R_1C + 1/LC)}$$

$$= \frac{4V \quad R_1C s^2}{s\cdot R_1C(s^2 + s/R_1C + 1/LC)}$$

$$= \frac{4V\cdot s}{s^2 + \frac{1}{R_1C}s + \frac{1}{LC}}$$

To match this to the symbolic form of $V_1(s)$ given earlier, we must have the same denominator:

$$s^2 + \frac{1}{R_1C}s + \frac{1}{LC} = (s+\alpha)^2 + \beta^2$$

$$\shortparallel \qquad = s + 2\alpha s + \alpha^2 + \beta^2$$

Note: We must also match numerators,
$4V\cdot s = V_m\cos(\varphi)(s+\alpha) - V_m\sin(\varphi)\beta$,
but this will be possible given arbitrary V_m and φ.

By matching the denominators, we have the following eq'ns:

$$\frac{1}{R_1C} = 2\alpha \qquad \text{where} \quad \alpha = 100 \text{ rad/s}$$
$$C = 500\mu F$$

$$\frac{1}{LC} = \alpha^2 + \beta^2 \qquad \text{where} \quad \beta = 100 \text{ rad/s}$$

Solving for R_1 and L, we have the following:

$$R_1 = \frac{1}{2\alpha \cdot C} = \frac{1}{2(100)\cdot 500\mu}\,\Omega = 10\,\Omega$$

$$L = \frac{1}{C(\alpha^2+\beta^2)} = \frac{1}{500\mu(100^2+100^2)}\,H = 100\,mH$$