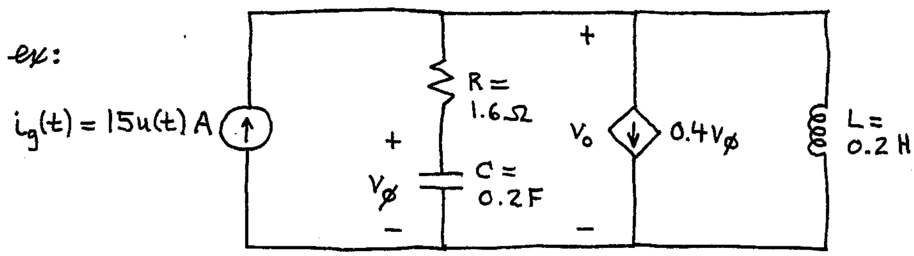
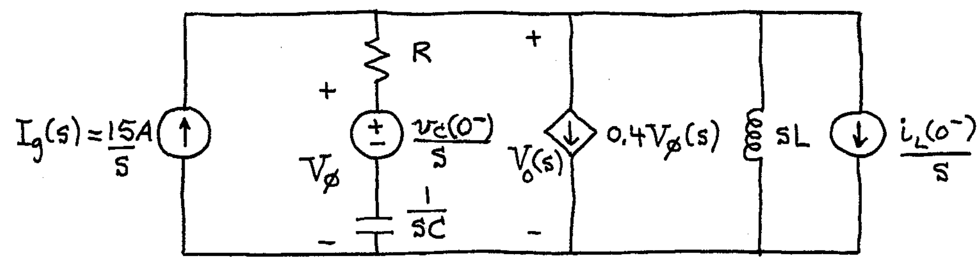
ex:



Find $v_o(t)$. $v_c(0^-) = 12V$, $i_L(0^-) = 5A$ flowing down

ans: $v_o(t) = -51.6te^{-5t} + 20.32e^{-5t}$ V

soln: We transform the circuit to the s-domain by
1) taking the Laplace transform of $i_g(t)$,
2) taking " " " " L and C, and
3) including sources for initial condition in L and C.

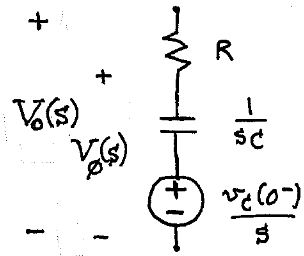


At this point, we may use Node-voltages, current meshes, Thevenin equivalents, or any other method that we would apply when solving a problem involving impedances.

Superposition is particularly useful since we may find the response of the circuit to each source, including those for initial conditions.

Since we have a dependent source, we first define V_ϕ in terms of node voltage(s).

We rearrange the branch containing V_ϕ:



The voltage across R and $\frac{1}{sC}$ is $V_o(s) - \frac{v_c(0^-)}{s}$. We use this voltage in the voltage divider formula to find the voltage across $\frac{1}{sC}$:

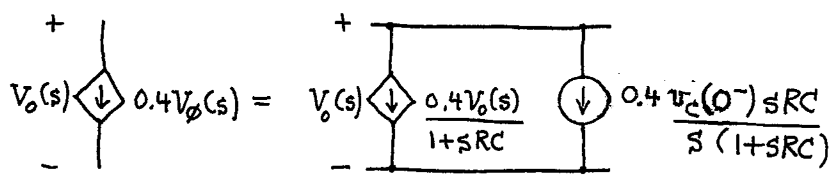
$$V_c(s) = \left[V_o(s) - \frac{v_c(0^-)}{s}\right] \frac{1/sC}{R + 1/sC}$$

Adding $V_c(s)$ to $\frac{v_c(0^-)}{s}$ gives $V_\phi(s)$:

$$V_\phi(s) = V_o(s)\frac{1/sC}{R + 1/sC} + \frac{v_c(0^-)}{s}\left(\frac{-1/sC}{R + 1/sC} + 1\right)$$

or $$V_\phi(s) = V_o(s)\frac{1}{1 + sRC} + \frac{v_c(0^-)}{s}\frac{sRC}{1 + sRC}$$

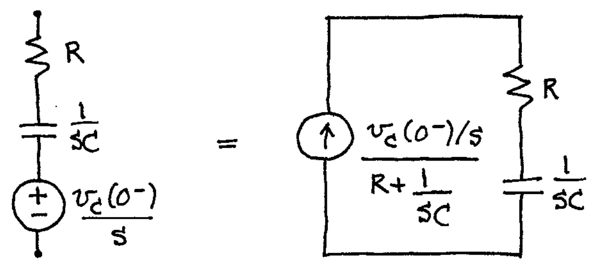
At this point, we observe that we may represent the dependent current source as two current sources that sum to produce current $0.4V_\phi(s)$:



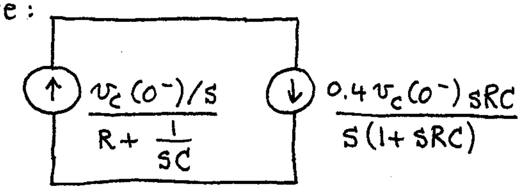
The leftmost of the two sources is equivalent to an impedance, $Z = V/I$:

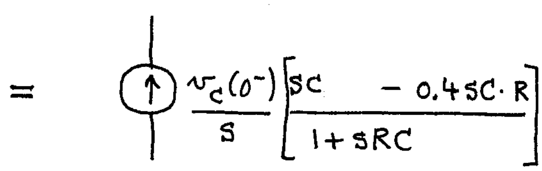
$$Z = \frac{V_o(s)}{\frac{0.4V_o(s)}{1 + sRC}} = \frac{1 + sRC}{0.4}\,\Omega$$

Because we already have four current sources in parallel, it is convenient to convert the branch containing the C from its Thevenin form to a Norton equivalent form.

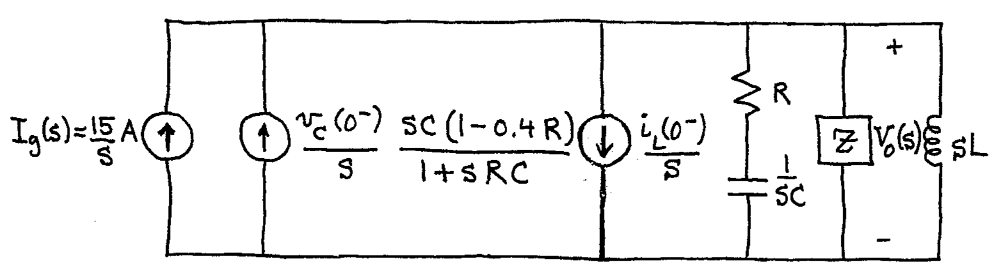


We may combine the current source from this Norton equivalent with the indepent source from the equivalent circuit for the dependent source:





Combining all of the above results and putting current sources together, we have a final model:



Now we have $V_o(s) = I_{Tot}(s) \cdot Z_{Tot}(s)$.

We calculate Z_{Tot} using the following identity for impedances written as $z_i(s) = b_i(s)/a_i(s)$ where $b_i(s)$ and $a_i(s)$ are polynomials in s:

$$z_1 \| z_2 \| \cdots \| z_N = \frac{b_1(s)}{a_1(s)} \Bigg\| \frac{b_2(s)}{a_2(s)} \Bigg\| \cdots \Bigg\| \frac{b_N(s)}{a_N(s)}$$

$$= b_1(s) \cdot b_2(s) \cdot \ldots \cdot b_N(s)$$

$$\cdot \frac{1}{a_1(s) b_2(s) \ldots b_N(s)} \Bigg\| \frac{1}{a_2(s) b_1(s) b_3(s) \ldots b_N(s)}$$

$$\Bigg\| \cdots \Bigg\| \frac{1}{a_N(s) b_1(s) \ldots b_{N-1}(s)}$$

$$= \frac{b_1(s)\, b_2(s) \ldots b_N(s)}{a_1(s) b_2(s) \ldots b_N(s) + a_2(s) b_1(s) b_3(s) \ldots b_N(s) + \ldots + a_N(s)\, b_1(s) \ldots b_{N-1}(s)}$$

Note: This identity is based on conductance and the observation that, e.g.,

$$\frac{1}{a} \Big\| \frac{1}{b} \Big\| \frac{1}{c} = \frac{1}{a+b+c}$$

In our case, we have $z_1 = R + \frac{1}{sC} = \frac{1+sRC}{sC}$

$$Z \equiv z_2 = \frac{1+sRC}{0.4}\ \Omega$$

$$z_3 = sL = \frac{sL}{1}$$

Using the identity, $\underbrace{z_1 \| z_2 \| z_3}_{z_{Tot}} = \dfrac{(1+sRC)^2 sL}{sC(1+sRC)sL + 0.4(1+sRC)sL + (1+sRC)^2}$

After cancellation of $1+sRC$:

$$Z_{Tot} = \frac{(1+sRC)\,sL}{s^2LC + 0.4sL + (1+sRC)}$$

Thus, we have

$$V_o(s) = I_{Tot}(s)\,Z_{Tot}(s) = \left(\frac{15}{s}A + \frac{i_L(0^-)}{s} + \frac{v_C(0^-)}{s}\,\frac{sC(1-0.4R)}{1+sRC}\right) \cdot \frac{(1+sRC)\,sL}{s^2LC + 0.4sL + 1 + sRC}$$

$$= 15A \cdot \frac{L(1+sRC)}{s^2LC + 0.4sL + 1 + sRC}$$

$$- i_L(0^-)\,\frac{L(1+sRC)}{s^2LC + 0.4sL + 1 + sRC}$$

$$+ v_C(0^-)\,\frac{C(1-0.4R)\,sL}{s^2LC + 0.4sL + 1 + sRC}$$

Note: we retain the terms for each original source and initial condition.

Now we divide the numerators and denominators by LC to get denominators with the coefficient of s^2 equal to unity.

$$V_o(s) = \frac{15A\;(1+sRC)/C}{s^2 + (0.4/C + R/L)s + 1/LC}$$

$$- \frac{i_L(0^-)\,(1+sRC)/C}{s^2 + (0.4/C + R/L)s + 1/LC}$$

$$+ \frac{v_C(0^-)\,(1-0.4R)\,s}{s^2 + (0.4/C + R/L)s + 1/LC}$$

We find roots of the denominators:

$$s^2+(0.4/C+R/L)s+1/LC = s^2+(0.4/0.2+1.6/0.2)s+1/(0.2)(0.2)$$

$$= s^2+10s+25 = (s+5)^2$$

Other numerical values we need:

$$1+sRC = 1+s(1.6)(0.2) = 1+s\cdot 8/25$$

$$1-0.4R = 1-0.4(1.6) = 9/25$$

We now use partial fractions.

$$\frac{(1+sRC)/C}{s^2+(0.4/C+R/L)s+1/LC} = \frac{1+s\cdot 8/25}{(s+5)^2(0.2)} = \frac{5K_1}{(s+5)^2} + \frac{5K_2}{s+5}$$

where $K_1 = \cancel{(s+5)^2}\left.\frac{(1+s\cdot 8/25)}{\cancel{(s+5)^2}}\right|_{s=-5} = 1-40/25 = -3/5$

$$K_2 = \left.\frac{d}{ds}(1+s\cdot 8/25)\right|_{s=-5} = \frac{8}{25}$$

For the $v_C(0^-)$ term:

$$\frac{(1-0.4R)s}{s^2+(0.4/C+R/L)s+1/LC} = \frac{(9/25)s}{(s+5)^2} = \frac{K_3}{(s+5)^2} + \frac{K_4}{s+5}$$

where $K_3 = \cancel{(s+5)^2}\left.\frac{(9/25)s}{\cancel{(s+5)^2}}\right|_{s=-5} = -9/5$

$$K_4 = \left.\frac{d}{ds}(9/25)s\right|_{s=-5} = 9/25$$

Using $i_L(0^-) = 5A$ and $v_C(0^-) = 12V$ we have a partial fraction expression for $V_o(s)$:

$$V_o(s) = 15A \cdot \left[\frac{-3}{(s+5)^2} + \frac{8/5}{s+5} \right]$$

$$-5A \left[\frac{-3}{(s+5)^2} + \frac{8/5}{s+5} \right]$$

$$+12V \left[\frac{-9/5}{(s+5)^2} + \frac{9/25}{s+5} \right]$$

We use $\mathcal{L}^{-1}\left\{ \frac{1}{(s+5)^2} \right\} = te^{-5t}$ and $\mathcal{L}^{-1}\left\{ \frac{1}{s+5} \right\} = e^{-5t}$

to find $v_o(t)$:

$$v_o(t) = -45te^{-5t} + 24e^{-5t}$$

$$+15te^{-5t} - 8e^{-5t}$$

$$-21.6te^{-5t} + 4.32e^{-5t} \quad V$$