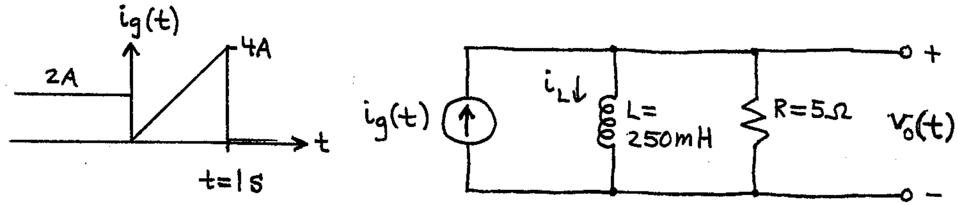
ex:

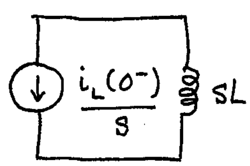


Find $v_o(t)$.

ans:

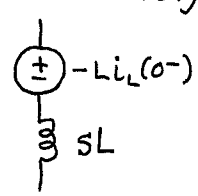
soln: Since the source, $i_g(t)$, has a constant value for a long time before $t=0$, L will act like a wire at $t=0^-$. Thus, all of the current from $i_g(t)$ will flow thru the L at $t=0^-$, and $i_L(0^-) = 2A$.

Our s-domain model for L:



$i_L(0^-) = 2A$

Note: We choose this model for convenience, (in that we may combine this current source with $i_g(t)$). We could have chosen an alternative model consisting of a voltage source in series with sL:



Our next step is to find the Laplace transform of $i_g(t)$.

For $t>0$, we have $i_g(t) = 4At\left[u(t)-u(t-1)\right]$

We write $i_g(t)$ in a form that allows us to use the time-delay identity for Laplace transforms:

$$\mathcal{L}\{f(t-a)\,u(t-a) \text{ where } a>0\} = e^{-as}\mathcal{L}\{f(t)u(t)\}$$

We proceed to find a way to write $-4At \cdot u(t-1)$ as a function of $(t-1)$:

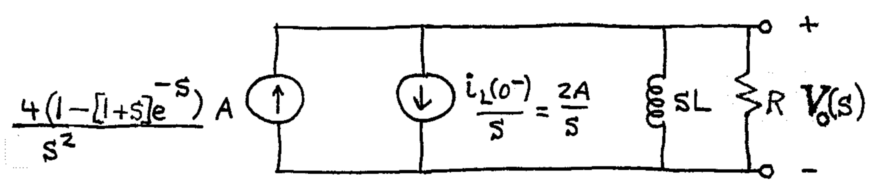
$$\begin{aligned} -4At\cdot u(t-1) &= -4A(t-1+1)\cdot u(t-1) \\ &= -4A[(t-1)+1]\,u(t-1) \\ &\equiv f(t-1)\,u(t-1) \end{aligned}$$

$$\text{where } f(t-1) = -4A[(t-1)+1]$$
$$\therefore\ f(t) = -4A[t+1]$$

Thus, $\mathcal{L}\{i_g(t)\} \equiv I_g(s) = \mathcal{L}\{4At\,u(t)\} + e^{-s}\mathcal{L}\{-4A(t+1)u(t)\}$

$$\text{or } I_g(s) = 4A\left(\frac{1}{s^2} - e^{-s}\left[\frac{1}{s^2}+\frac{1}{s}\right]\right)$$

Combining results gives us the s-domain model:



We have $V_o(s) = I_{Tot}(s)\cdot sL \| R = I_{Tot}(s)\cdot \dfrac{s\cdot R}{s+R/L}$

$$\text{where } I_{Tot}(s) = \frac{4A(1-[1+s]e^{-s})}{s^2} - \frac{2A}{s}$$

Multiplying thru, we have the following for $V_o(s)$:

$$V_o(s) = 4A\frac{(1-[1+s]e^{-s})R}{s(s+R/L)} - 2A\frac{R}{s+R/L}$$

Using $R = 5\,\Omega$ and $R/L = 5\,\Omega/250\,\text{mH} = 20\ s^{-1}$:

$$V_o(s) = \frac{20\,V(1-e^{-s})}{s(s+20)} - \frac{20\,Ve^{-s}}{s+20} - \frac{10\,V}{s+20}$$

For the first term, we use partial fractions:

$$\frac{1}{s(s+20)} = \frac{K_1}{s} + \frac{K_2}{s+20}$$

where $K_1 = \cancel{s}\cdot\frac{1}{\cancel{s}(s+20)}\Big|_{s=0} = \frac{1}{20}$

$$K_2 = \cancel{(s+20)}\frac{1}{s\cancel{(s+20)}}\Big|_{s=-20} = -\frac{1}{20}$$

$$\therefore V_o(s) = \frac{1\,V(1-e^{-s})}{s} - \frac{1\,V(1-e^{-s})}{s+20} - \frac{20\,Ve^{-s}}{s+20} - \frac{10\,V}{s+20}$$

From the delay identity, $\mathcal{L}^{-1}\left\{\frac{e^{-s}}{s+a}\right\} = e^{-a(t-1)}u(t-1)$.

$$\therefore v_o(t) = 1V\,u(t) - 1V\,u(t-1)$$
$$-1Ve^{-20t}u(t) + 1Ve^{-20(t-1)}u(t-1)$$
$$-20V\,e^{-20(t-1)}u(t-1) - 10Ve^{-20t}u(t)$$