EX: Find $\mathcal{L}\left\{tu(t-3)e^{-4t}\right\}$.

SOL'N: We use the time shift identity:

$$\mathcal{L}\{f(t-a)u(t-a)\} = e^{-as}F(s)$$

The time-shifted step function, $u(t-3)$, dictates that the time delay is $a = 3$. To use the identity, we find a way to write the entire expression being transformed into a function of $t - 3$. We achieve this by subtracting and adding 3 from and to t:

$$\mathcal{L}\left\{tu(t-3)e^{-4t}\right\} = \mathcal{L}\left\{([t-3]+3)u(t-3)e^{-4([t-3]+3)}\right\}$$

By replacing $t - 3$ with t throughout, we find $f(t)$:

$$f(t) = (t+3)e^{-4(t+3)}$$

Now we Laplace transform $f(t)$:

$$F(s) = \mathcal{L}\left\{te^{-4(t+3)} + 3e^{-4(t+3)}\right\}$$

or

$$F(s) = \mathcal{L}\left\{te^{-12}e^{-4t} + 3e^{-12}e^{-4t}\right\}$$

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$$F(s) = e^{-12}\mathcal{L}\left\{te^{-4t}\right\} + 3e^{-12}\mathcal{L}\left\{e^{-4t}\right\}$$

or

$$F(s) = e^{-12}\frac{1}{(s+4)^2} + 3e^{-12}\frac{1}{s+4}$$

Applying the identity yields the final answer:

$$\mathcal{L}\left\{tu(t-3)e^{-4t}\right\} = e^{-3s}\left[e^{-12}\frac{1}{(s+4)^2} + 3e^{-12}\frac{1}{s+4}\right]$$