

TOOL: The following list of identities allows one to generate complicated Laplace transforms from basic Laplace transforms. Proofs follow from the definition of the Laplace transform:

$$\mathcal{L}[v(t)] \equiv V(s) \equiv \int_{0^-}^{\infty} v(t)e^{-st} dt$$

Other terms used are $\delta(t)$, which is the impulse (or delta) function, and $u(t)$, which is the unit step function.

$$\delta(t) \equiv \begin{cases} 0 & t \neq 0 \\ \infty & t = 0 \end{cases} \quad \text{with} \quad \int_{-\infty}^{\infty} \delta(t)dt = 1, \quad u(t) \equiv \begin{cases} 0 & t < 0 \\ 1 & t \geq 0 \end{cases}$$

| LIST: | Name | <i>t</i> -domain | <i>s</i> -domain | Condition |
|-------|------------------------|------------------------|--|------------|
| | definition | $v(t)$ | $V(s)$ | |
| | linearity | $av_1(t) + bv_2(t)$ | $aV_1(s) + bV_2(s)$ | |
| | delay | $v(t - a)u(t - a)$ | $e^{-as}V(s)$ | $a \geq 0$ |
| | multiply by <i>t</i> | $tv(t)$ | $-\frac{d}{ds}V(s)$ | |
| | multiply by e^{-at} | $e^{-at}v(t)$ | $V(s + a)$ | $a \geq 0$ |
| | divide by <i>t</i> | $\frac{v(t)}{t}$ | $\int_s^{\infty} V(s)ds$ | |
| | derivative | $\frac{d}{dt}v(t)$ | $sV(s) - v(t=0^-)$ | |
| | <i>n</i> th derivative | $\frac{d^n}{dt^n}v(t)$ | $s^nV(s) - s^{n-1}v(t)\Big _{t=0^-}$ $-s^{n-2}\frac{d}{dt}v(t)\Big _{t=0^-}$ \vdots $-s^0\frac{d^{n-1}}{dt^{n-1}}v(t)\Big _{t=0^-}$ | |
| | integral | $\int_{0^-}^t v(t)dt$ | $\frac{V(s)}{s}$ | |
| | time scaling | $v(at)$ | $\frac{1}{a}V\left(\frac{s}{a}\right)$ | $a \geq 0$ |

REF: James A. Nilsson, Susan A. Riedel, *Electric Circuits*, 8th Ed., Upper Saddle River, NJ: Prentice Hall, 2007.