

EX: Find

$$\lim_{t \rightarrow \infty} f(t) \text{ if } F(s) = \frac{3}{s[(s+4)^2 + 36]}$$

SOL'N: Apply the final value theorem:

$$\lim_{t \rightarrow \infty} f(t) = \lim_{s \rightarrow 0} s \mathcal{L}\{f(t)\} = \lim_{s \rightarrow 0} sF(s)$$

We first factor out the highest power of s from the numerator and denominator and cancel out as many powers of s as possible:

$$sF(s) = \frac{s}{s} \cdot \frac{3}{[(s+4)^2 + 36]} = \frac{3}{[(s+4)^2 + 36]}$$

If there are pure powers of s remaining in the numerator or denominator, we may immediately conclude that the answer is zero or infinity, respectively.

Otherwise, as in the present case, we proceed to substitute $s = 0$ in the numerator and denominator to obtain our final result:

$$\lim_{t \rightarrow \infty} f(t) = \lim_{s \rightarrow 0} sF(s) = \frac{3}{4^2 + 36} = \frac{3}{52}$$