EX: Find the inverse Laplace transform for the following expression:

$$F(s) = \frac{9s + 15}{s^2 + 5s}$$

SOL'N: First, we factor the denominator:

$$F(s) = \frac{9s + 15}{s(s + 5)}$$

Second, we write $F(s)$ in terms of partial fractions:

$$F(s) = \frac{K_1}{s} + \frac{K_2}{s + 5}$$

Third, we find the coefficients for the partial fractions by multiplying the root (or pole) term and setting s equal to the root:

$$K_1 = sF(s)\Big|_{s=0} = s\frac{9s + 15}{s(s + 5)}\Bigg|_{s=0} = \frac{9s + 15}{(s + 5)}\Bigg|_{s=0} = \frac{15}{5} = 3$$

The above calculation shows that we cancel a root (or pole) term in the denominator when we multiply the numerator by that root term. We save a step in the following calculation by canceling the root term:

$$K_2 = (s + 5)F(s)\Big|_{s=-5} = \frac{9s + 15}{s}\Bigg|_{s=-5} = \frac{-30}{-5} = 6$$

Fourth, we take the inverse Laplace transform of each term using the following basic identity:

$$\mathcal{L}^{-1}\left\{\frac{K}{s + a}\right\} = Ke^{-at}$$

Note that we treat s as $s + 0$.