Linear Algebra—

20 Apr 1989 Graham - Schmidt in N dimensions

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Assume $\vec{x}_1$ is given. Let $\vec{r}_2, \ldots, \vec{r}_N$ be chosen at random.

Find $\vec{x}_2, \ldots, \vec{x}_N$ such that $\vec{x}_i \circ \vec{x}_j = 0$ for all $i \neq j$ (i.e. $\vec{x}_i \perp \vec{x}_j$)

Algorithm: $$\vec{x}_2 = \left(\vec{r}_2 - P_{x_1}\vec{r}_2\right) / \left|\vec{r}_2 - P_{x_1}\vec{r}_2\right| = \frac{\perp_{x_1}\vec{r}_2}{\left|\perp_{x_1}\vec{r}_2\right|}$$

$$= \frac{\vec{r}_2 - P_{x_1}\vec{r}_2}{\left|\vec{r}_2 - P_{x_1}\vec{r}_2\right|}$$

For $\vec{x}_3$ we must subtract out the projection on $\vec{x}_1$ <u>and</u> the projection on $\vec{x}_2$ so we will have $\vec{x}_2 \perp \vec{x}_3$, too.

$$\vec{x}_3 = \frac{\vec{r}_3 - P_{x_1}\vec{r}_3 - P_{x_2}\vec{r}_3}{\left|\vec{r}_3 - P_{x_1}\vec{r}_3 - P_{x_2}\vec{r}_3\right|}$$

Careful analysis of this process reveals that this only works when $\vec{x}_1$ and $\vec{x}_2$ are already orthogonal (perpendicular) to each other.

Now the pattern continues

$$\vec{x}_4 = \frac{\vec{r}_4 - P_{x_1}\vec{r}_4 - P_{x_2}\vec{r}_4 - P_{x_3}\vec{r}_4}{\left|\vec{r}_4 - P_{x_1}\vec{r}_4 - P_{x_2}\vec{r}_4 - P_{x_3}\vec{r}_4\right|}$$

etc.