EX: Equations for the field-oriented control scheme of an induction motor based on the DQ model are as follows, [1]:

$$\frac{d\psi}{dt} = -\frac{1}{T_R}\psi + \frac{M}{T_R}i_{sd}$$

$$\frac{d\omega}{dt} = \frac{n_p M}{JL_R}\psi i_{sq} - \frac{B\omega}{J}$$

where

$\psi \equiv$ rotor flux (Webers)
$T_R \equiv$ rotor time constant $= L_R/R_R$ (s)
$L_R \equiv$ rotor inductance (H)
$R_R \equiv$ rotor resistance (Ω)
$M \equiv$ mutual inductance of rotor and stator coils (H)
$\omega \equiv$ speed of motor (rad/s)
$J \equiv$ moment of inertia of motor (kg m^2)
$B \equiv$ dynamic friction constant (Nms)

Motor parameters:

$$L_R = \frac{2}{100}\text{H} \qquad R_R = \frac{2}{10}\Omega \qquad M = \frac{1}{100}\text{H}$$

$$J = 0.05\ \text{kgm}^2 \qquad B = 20\text{Nms} \qquad n_p = 1$$

$$i_{sq0} = 100\text{A} \qquad \psi_0 = 4\text{Tm}^2$$

a) Linearize the motor equations around the fixed point for the above motor parameters. (Parameters with subscript 0 represent values for the fixed point.) Treat i_{sd} and i_{sq} as inputs with small variations around the fixed point. Give the numerical values of entries in the matrices A and B in the linearized equation $dx_\varepsilon/dt = Ax_\varepsilon + Bi_\varepsilon$ where $x_\varepsilon \equiv [\psi, \omega]^T$ and $i_\varepsilon \equiv [i_{sd}, i_{sq}]$.

b) Now assume that $\psi = \psi_0$ is constant. Use the second linearized equation alone and assume PI control of the following form:

$$i_{sq\varepsilon} = k_P(k_F\omega_{ref\varepsilon} - \omega_\varepsilon) + k_I\int_0^t(\omega_{ref\varepsilon} - \omega_\varepsilon)dt$$

(where $\omega_{ref\varepsilon} \equiv \omega_{ref} - \omega_0$). Find the transfer function $\Omega_\varepsilon/\Omega_{ref\varepsilon}$ for the system. Also, draw a root locus plot for the location of poles versus k_P. Assume $k_I/k_P = 10$.

REF: [1] Marc Bodson, "Control of Electric Motors," 2004, University of Utah ECE Dept., eqn 4.107 p. 152.

Sol'n: a) We use the Jacobian matrices for the motor eq'ns.

$$\begin{bmatrix} \frac{d\psi}{dt} \\ \frac{d\omega}{dt} \end{bmatrix} = \begin{bmatrix} \frac{d(\psi_0+\psi_\epsilon)}{dt} \\ \frac{d(\omega_0+\omega_\epsilon)}{dt} \end{bmatrix} = \begin{bmatrix} \frac{d\psi_\epsilon}{dt} \\ \frac{d\omega_\epsilon}{dt} \end{bmatrix} \doteq \overset{A}{\overset{\shortparallel}{\left. \begin{bmatrix} \frac{\partial f_1}{\partial \psi} & \frac{\partial f_1}{\partial \omega} \\ \frac{\partial f_2}{\partial \psi} & \frac{\partial f_2}{\partial \omega} \end{bmatrix} \right|_{\psi=\psi_0,\ \omega=\omega_0}}} \begin{bmatrix} \psi_\epsilon \\ \omega_\epsilon \end{bmatrix}$$

$$+ \underset{B}{\underset{\shortparallel}{\left. \begin{bmatrix} \frac{\partial f_1}{\partial i_{sd}} & \frac{\partial f_1}{\partial i_{sq}} \\ \frac{\partial f_2}{\partial i_{sd}} & \frac{\partial f_2}{\partial i_{sq}} \end{bmatrix} \right|_{\psi=\psi_0,\ \omega=\omega_0}}} \begin{bmatrix} i_{sd\epsilon} \\ i_{sq\epsilon} \end{bmatrix}$$

where $f_1 \equiv -\frac{1}{T_R}\psi + \frac{M}{T_R} i_{sd}$ (eq'n for $\frac{d\psi}{dt}$)

$f_2 \equiv \frac{n_p M}{J L_R} \psi i_{sq} - \frac{B\omega}{J}$ (eq'n for $\frac{d\omega}{dt}$)

Using the notation given in the problem, we write the above system eq'n for small perturbations as

$$\frac{dx_\epsilon}{dt} = A x_\epsilon + B i_\epsilon \qquad \text{where } x_\epsilon \equiv [\psi, \omega]^T$$

Taking derivatives for the Jacobians gives

$$A = \begin{bmatrix} -\frac{1}{T_R} & 0 \\ \frac{n_p M}{J L_R} i_{sq0} & -\frac{B}{J} \end{bmatrix}, \quad B = \begin{bmatrix} \frac{M}{T_R} & 0 \\ 0 & \frac{n_p M \psi_0}{J L_R} \end{bmatrix}$$

$$\text{or} \quad A = \begin{bmatrix} -10 & 0 \\ 1000 & -500 \end{bmatrix}, \quad B = \begin{bmatrix} \frac{1}{10} & 0 \\ 0 & 40 \end{bmatrix}$$

b) In the s-domain, our PI control is

$$I_{sqe}(s) = k_p\left(\Omega_{ref}(s) - \Omega(s)\right) + \frac{k_I}{s}\left(\Omega_{ref}(s) - \Omega(s)\right)$$

Our linearized eq'n for the motor (with $\psi_\epsilon = 0$) becomes

$$\frac{d\omega_\epsilon}{dt} = -\frac{B}{J}\omega_\epsilon + \frac{n_p M \Psi_o}{J L_R} i_{sq\epsilon}$$

$$\text{or} \quad \frac{d\omega_\epsilon}{dt} = -500\,\omega_\epsilon + 40\, i_{sq\epsilon}$$

In the s-domain, we have

$$s\Omega(s) = -\frac{B}{J}\Omega(s) + \frac{n_p M \Psi_o}{J L_R} I_{sq\epsilon}(s)$$

$$\text{or} \quad \Omega(s) = \underbrace{\frac{n_p M \Psi_o}{J L_R}}_{\equiv K} \frac{1}{s + \frac{B}{J}} I_{sq\epsilon}(s)$$

Combining this eq'n with the control law gives

$$\Omega(s) = \frac{K}{s + \frac{B}{J}}\, k_p\left(\Omega_{ref}(s) - \Omega(s)\right) + \frac{k_I}{s}\left(\Omega_{ref} - \Omega(s)\right)$$

$$= \frac{K}{s + \frac{B}{J}}\left(k_p + \frac{k_I}{s}\right)\Omega_{ref}(s)$$

$$- \frac{K}{s + \frac{B}{J}}\left(k_p + \frac{k_I}{s}\right)\Omega(s)$$

Solving for $\dfrac{\Omega(s)}{\Omega_{ref}(s)}$ yields the following:

$$\frac{\Omega(s)}{\Omega_{ref}(s)} = \frac{\dfrac{K}{s+\frac{B}{J}}\left(k_P + \dfrac{k_I}{s}\right)}{1 + \dfrac{K}{s+\frac{B}{J}}\left(k_P + \dfrac{k_I}{s}\right)}$$

$$\frac{\Omega(s)}{\Omega_{ref}(s)} = \frac{K \cdot k_P \left(s + \dfrac{k_I}{k_P}\right)}{s\left(s+\frac{B}{J}\right) + K k_P\left(s + \dfrac{k_I}{k_P}\right)}$$

For the root-locus plot versus k_P with $\dfrac{k_I}{k_P} = 10$

we put the denominator in form $1 + k_P G(s)$:

$$1 + k_P \frac{K(s + k_I/k_P)}{s\left(s+\frac{B}{J}\right)} = 0$$

Using values given in the problem,

$$K = \frac{n_P M \Psi_0}{J L_R} = \frac{1 \cdot 10m \cdot 4}{50m\ 20m} \text{ per } As^2 = 40\ /As^2$$

$$\frac{B}{J} = \frac{20}{50m} = 400$$

Thus, $$1 + k_P \frac{40 \cdot (s+10)}{s(s+400)} = 0.$$