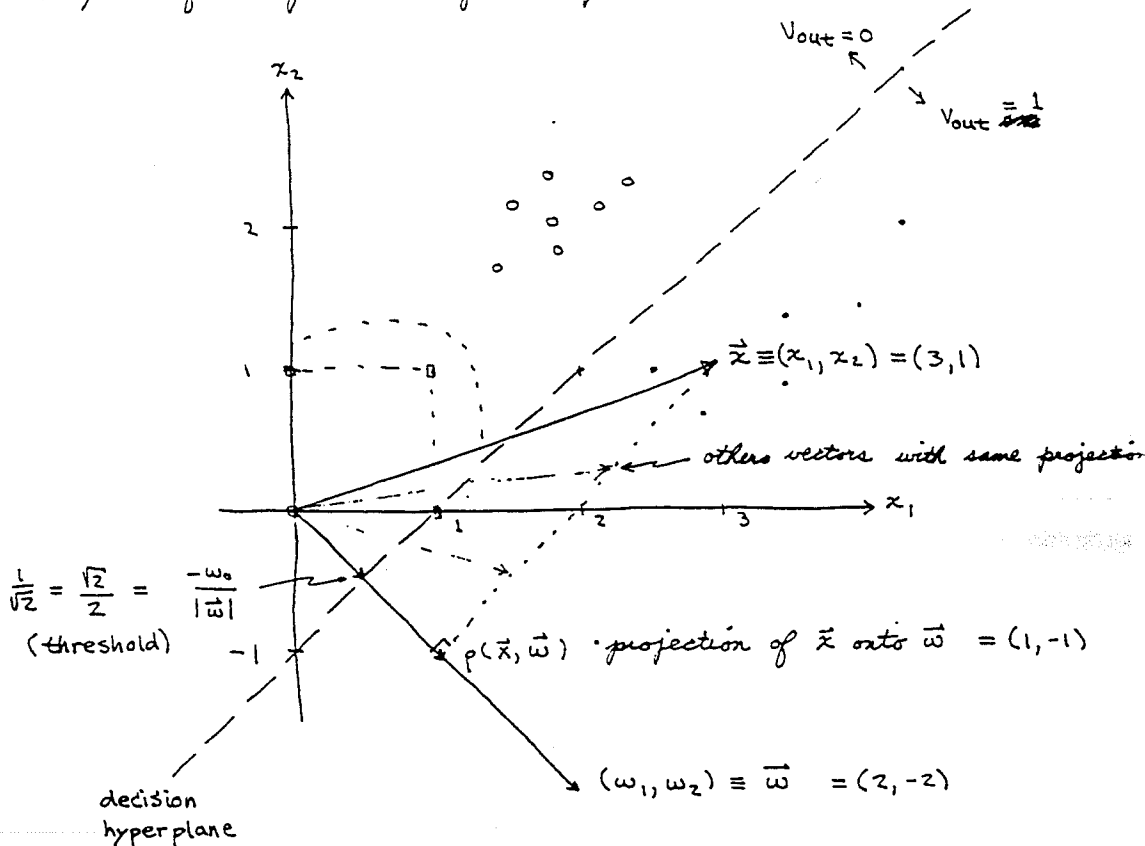


Perceptrons - Decision Geometry  
 Geometry of computation by Perception

24 April 1988  
 Neil E Cottler

ex:



- Observe:
- 1) decision line  $\perp$  weight vector
  - 2) " " offset from origin by  $\frac{-w_0}{|\vec{w}|}$
  - 3)  $V_{out} = 1$  when  $\underbrace{\text{length of projection of } \vec{x} \text{ onto } \vec{w}}_{p(\vec{x}, \vec{w})} > \frac{-w_0}{|\vec{w}|}$
  - 4) Can get any desired decision line by choosing  $\vec{w}$  and  $w_0$

Math:

$$V_{out} = 1 \text{ when } 1 \cdot w_0 + x_1 w_1 + x_2 w_2 \geq 0$$

$$x_1 w_1 + x_2 w_2 \geq -w_0$$

$$\vec{x} \cdot \frac{\vec{w}}{|\vec{w}|} \geq \frac{-w_0}{|\vec{w}|}$$

But  $\left( \frac{\vec{x} \cdot \vec{w}}{|\vec{w}|} \right) \frac{|\vec{w}|}{|\vec{w}|} \equiv p(\vec{x}, \vec{w})$

length of  $p(\vec{x}, \vec{w})$  vector  
 unit vector

def:  $\vec{x} \parallel \vec{w} \equiv \vec{x} \cdot \frac{\vec{w}}{|\vec{w}|}$  - Part of  $\vec{x}$  that is parallel to  $\vec{w}$

$\therefore V_{out} = 1$  when  $\vec{x} \parallel \vec{w} > -w_0 / |\vec{w}|$

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Perceptrons - Decision Geometry (cont.)

Neil E Cottar

• Use numbers from example

$$\vec{x} = (3, 1)$$

$$\vec{w} = (2, -2)$$

$$w_0 = -2$$

$$\vec{x} \cdot \vec{w} \equiv \vec{x} \cdot \frac{\vec{w}}{|\vec{w}|} = \frac{3 \cdot 2 + 1(-2)}{\sqrt{2^2 + (-2)^2}} = \frac{4}{2\sqrt{2}} = \frac{2}{\sqrt{2}}$$

$$-\frac{w_0}{|\vec{w}|} = \frac{-(-2)}{2\sqrt{2}} = \frac{1}{\sqrt{2}}$$

$$\frac{2}{\sqrt{2}} > \frac{1}{\sqrt{2}} \Rightarrow V_{\text{out}} = 1 \quad \checkmark$$