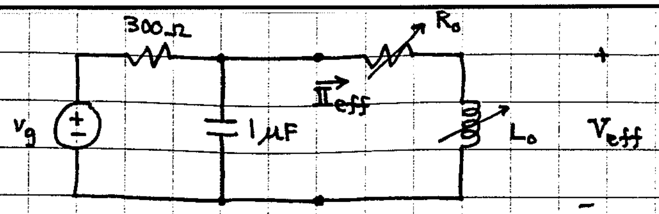


Peak amplitude of v_g is $150\sqrt{2}$ V with period 200π μs.

Note: $\omega = 2\pi f \quad f = 1/T \quad T = \text{period}$

$$\therefore \omega = \frac{2\pi}{T} = 10k/s \qquad \frac{2\pi}{2\pi \cdot 100\,\mu s} = 10k/s$$

a) Calculate the average power delivered to the load when $R_o = 10\,\Omega$ and $L_o = 6\,mH$.

soln: $S = P + jQ = V_{eff}\, I_{eff}^{*}$ where eff = rms

$*$ = complex conjugate

$$\left.\begin{array}{l} P = |I_{eff}|^2 R \\ Q = |I_{eff}|^2 X \end{array}\right\} \text{ for } z = R + jX$$

So we need to find I_{eff}.

$$I_{eff} = \frac{V_{eff}}{R_o + j\omega L_o}$$

$$V_{eff} = V_{g\,eff} \cdot \frac{(R_o + j\omega L_o) \,\|\, -j/\omega C}{R_o + j\omega L_o \,\|\, -j/\omega C + 300\,\Omega}$$

$$V_{eff} = \frac{V_{g\,eff}\ (R_o + j\omega L_o) \cdot \dfrac{-j/\omega C}{R_o + j\omega L_o - j/\omega C}}{(R_o + j\omega L_o) \cdot \dfrac{-j/\omega C}{R_o + j\omega L_o - j/\omega C} + 300\,\Omega}$$

$$\therefore I_{eff} = \frac{V_{g\,eff}\ \dfrac{-j/\omega C}{R_o + j\omega L_o - j/\omega C}}{(R_o + j\omega L_o)\dfrac{(-j/\omega C)}{R_o + j\omega L_o - j/\omega C} + 300\,\Omega}$$

mult top & bottom by $R_o + j(\omega L_o - 1/\omega C)$

$$\mathbb{I}_{\text{eff}} = \frac{V_{g\,\text{eff}}\,(-j/\omega C)}{(R_o + j\omega L_o)(-j/\omega C) + 300\,\Omega\,(R_o + j\omega L_o - j/\omega C)}$$

$$\mathbb{I}_{\text{eff}} = \frac{V_{g\,\text{eff}}}{R_o + j\omega L_o + 300\,\Omega\left(\dfrac{\omega C R_o}{-j} + \dfrac{j\omega L_o \omega C}{-j} + 1\right)}$$

$$\mathbb{I}_{\text{eff}} = \frac{V_{g\,\text{eff}}}{R_o + j\omega L_o + 300\,\Omega\left(j\omega C R_o - \omega^2 L_o C + 1\right)}$$

We can use this general formula for $R_o = 10\,\Omega$, $L_o = 6\,\text{mH}$.

$\omega L_o = 10\text{k} \cdot 6\text{m} = 60\,\Omega$ $\qquad \omega^2 L_o C = 60\,\Omega \cdot 10\text{m}/\Omega = 0.6$

$\omega C = 10\text{k} \cdot 1\mu = 10\text{m}/\Omega$

$$V_{g\,\text{eff}} = \frac{150\sqrt{2}}{\sqrt{2}} = 150\text{ V (rms)}$$

$$\therefore\ \mathbb{I}_{\text{eff}} = \frac{150\text{ V(rms)}}{10 + j\,60 + 300\,(j\,10\text{m}\cdot 10 - 0.6 + 1)}$$

$$= \frac{150\text{ V(rms)}}{10 - 180 + 300 + j(60 + 30)}$$

$$= \frac{150\text{V (rms)}}{130 + j\,90}$$

$$P = |\mathbb{I}_{\text{eff}}|^2 \cdot R_o = \frac{|150\text{ V(rms)}|^2}{|130 + j\,90|^2} \cdot 10\,\Omega$$

$$= \frac{|15|^2}{|13 + j9|^2} \cdot 10\text{ W} = \frac{225 \cdot 10\text{W}}{169 + 81} = \frac{225 \cdot 10}{250} = 9\text{ W}$$

Optional:

b) Determine R_o and L_o for max power being transferred to R_o if $0 \le R_o \le 20\,\Omega$ and $1 \le L_o \le 8\,\text{mH}$.

sol'n: max power transfer when $Z_L = Z_{Th}^*$.

If that is not possible, we make X_L as close to $-X_{Th}$ as possible, and (after that is done) we make R_L as close to $\sqrt{R_{Th}^2 + (X_L + X_{Th})^2}$ as possible.

We find Thevenin equivalent of circuit with no load.

$$V_{Th} = \text{open circuit } V = V_g \cdot \frac{-j/\omega C}{-j/\omega C + 300\,\Omega}$$

Use rms V from now on:

$$V_{Th\,eff} = V_{g\,eff} \frac{1}{1 + j\omega C \cdot 300\,\Omega} = \frac{150 \text{ V (rms)}}{1 + j\,10\text{m}/\Omega \cdot 300\,\Omega} = \frac{150}{1+j3} \text{ V (rms)}$$

z_{Th} = z seen looking in from z_L terminals with $V_g = 0$

$$= 300 \,\|\, -j/\omega C = \frac{300 \cdot -j/10\text{m}}{300 - j/10\text{m}}\,\Omega = \frac{300}{\frac{10\text{m}}{-j} \cdot 300 + 1}$$

$$= \frac{300}{1+j3}\,\Omega = \frac{300(1-j3)}{10} = 30(1-j3) = 30 - j90\,\Omega$$

$$z_{Th} = 30 - j90\,\Omega$$

$$R_{Th} = 30 \qquad -X_{Th} = 90\,\Omega$$

So we desire $\omega L = 90\,\Omega$ or $L = \frac{90}{\omega} = \frac{90}{10\text{k}} = 9\text{ mH}$

Closest allowed value is 8 mH for L_o.

Then we want $R_L = \sqrt{R_{Th}^2 + (X_L + X_{Th})^2}$

$$(X_L + X_{Th})^2 = (\omega \cdot 8\text{ mH} + -90)^2 = (-10)^2 = 100\,\Omega^2$$

want $R_L = \sqrt{30^2 + 100} = \sqrt{1\text{K}} = 31.6\,\Omega$

Closest allowed value is 20 Ω for R_L.

c) Find ave power for (b). Compare to answer for (a).

sol'n: $\omega L_o = 80\,\Omega \qquad \omega C R_o = 10\text{m}/\Omega \cdot 20\,\Omega = 0.2$

$$\omega^2 L_o C = 80 \cdot 10m = 0.8$$

$$\therefore \mathbb{I}_{eff} = \frac{150\text{ V (rms)}}{20 + j80 + 300(j0.2 - 0.8 + 1)} \quad \text{(using formula from part 'a')}$$

$$= \frac{150\text{V (rms)}}{20 + 60 + j(80 + 60)} = \frac{150\text{V (rms)}}{80 + j140} = \frac{15\text{V (rms)}}{8 + j14}$$

$$P = |\mathbb{I}_{eff}|^2 \cdot R_o = \frac{|15|^2}{|8 + j14|^2} \cdot 20\text{ W} = \frac{225}{260} \cdot 20\text{ W}$$

$$P = \frac{225\text{W}}{13} = 17.3\text{ W} \quad \text{which is} > 9\text{W from (a).}$$

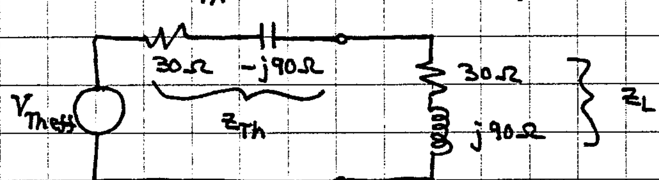
We expected larger P since R_o, L_o were optimized within ranges given, and R_o, L_o for part 'a' were in those ranges, too.

d) If no constraints on R_o, L_o, find optimal R_o, L_o pwr xfer.

sol'n: We want $z_L = z_{Th}^* = 30 + j90\,\Omega$

$\therefore$ $R_o = 30\,\Omega$ and $\omega L = 90\,\Omega \Rightarrow L = 9\text{ mH}$

With $z_L = z_{Th}^*$ we have equiv circuit:



$$\text{So } \mathbb{I}_{eff} = \frac{V_{Th\,eff}}{30 \cancel{- j90} + 30 \cancel{+ j90}\,\Omega} = \frac{150\text{ V(rms)}/(1+j3)}{60\,\Omega}$$

$$= \frac{5}{2(1+j3)}$$

$$P = |\mathbb{I}_{eff}|^2 R_o = \left(\frac{5}{2}\right)^2 \frac{1}{1^2 + 3^2} \cdot 30\text{ W} = 18.75\text{ W}$$

e) What are optimal R_o, L_o for max pwr xfer in (d)? sol'n: $R_o = 30\,\Omega$
see (d) $\rightarrow$ $L_o = 9\text{mH}$

f) Is pwr in (d) > pwr in (c)? Yes, as expected.