TUTORIAL: AC POWER

In a linear circuit with sinusoidal source of frequency ω, currents and voltages are sinusoids of frequency ω. Power is still calculated as $p = iv$. Thus, we multiply sinusoids, although there is typically a phase difference between the current and voltage. We define the phase difference as $(\theta_v - \theta_i)$. If we also define i_m and v_m as the amplitudes of current and voltage and adjust the origin of time so that $\theta_i = 0$, we have

$$p(t) = i(t)v(t) = i_m \cos(\omega t) \cdot v_m \cos(\omega t + \theta_v - \theta_i)$$

Note that we might try to leave the θ_i in the current term but the result is quite awkward to work with.

We apply a standard trigonometric identity to translate the product of sinusoids into a sum of sinusoids:

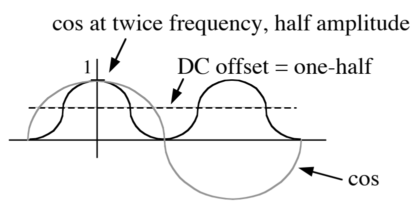
$$\cos A \cdot \cos B = \tfrac{1}{2}\cos(A - B) + \tfrac{1}{2}\cos(A + B)$$

where $A - B = \theta_v - \theta_i$ and $A + B = 2\omega t + \theta_v - \theta_i$.

The result is that the power has a constant (or DC) term (that is no longer dependent on time or frequency) and a sinusoidal signal (that has double the frequency of the current and voltage):

$$p(t) = \frac{i_m v_m}{2}\cos(\theta_v - \theta_i) + \frac{i_m v_m}{2}\cos(2\omega t + \theta_v - \theta_i)$$

Note also that there is a factor of one-half in both terms. A trick for remembering these features of the power waveform is to consider the power waveform when current and voltage are in phase. In that case, the product of i and v has the shape of $\cos^2(\omega t)$. Sketching $\cos^2(\omega t)$ reveals that it is the sum of $\cos(2\omega t)$ with amplitude one-half and a DC offset of one-half.



If we now think in terms of frequency 2ω instead of ω, we see that the second term of the power expression is a cosinusoid with a magnitude and phase offset. In other words, it is a sinusoidal signal represented in polar form. We may translate it into rectangular form consisting of a pure cosine and a pure sine:

$$\frac{i_m v_m}{2}\cos(2\omega t+\theta_v-\theta_i)=\frac{i_m v_m}{2}\cos(\theta_v-\theta_i)\cos(2\omega t)-\frac{i_m v_m}{2}\sin(\theta_v-\theta_i)\sin(2\omega t)$$

Note that this is only the sinusoidal (or AC) part of the power expression.

To simplify the notation, we define P and Q:

$$P=\frac{i_m v_m}{2}\cos(\theta_v-\theta_i)$$

$$Q=\frac{i_m v_m}{2}\sin(\theta_v-\theta_i)$$

By coincidence, P appears twice in the complete power expression, meaning we need only P and Q rather than three different terms:

$$p(t)=P+P\cos(2\omega t)-Q\sin(2\omega t)$$

Because we have both P and Q in the AC part of the power, (i.e., the last two terms),we achieve an economy of notation (and possibly a loss of clarity) by ignoring the DC part of the power and then using a phasor representation of the AC part:

$$S=P+jQ$$

Note that the sign is + for Q in the phasor, whereas the sign is – for Q in the expression for p. Also, this "complex power", S, happens to have, as its real part, the average or DC power P. Strictly speaking, however, the P represents the cosine part of the AC power.

If we use phasors for the original current and voltage waveforms, we may derive the following identities:

$$S=\tfrac{1}{2}\mathbf{I}^*\cdot\mathbf{V}=\mathbf{I}^*_{\text{rms}}\cdot\mathbf{V}_{\text{rms}}=\left|\mathbf{I}_{\text{rms}}\right|^2 Z$$

Once we have found S, we know P and Q and, hence, we know the complete power waveform, $p(t)$.