EX: Three engineers with initials L, C, and M have invented a cryptography scheme for sending 2 bits of information. Their idea is to pick a random number, X, from 1 to 4 by some unspecified method. According to the value of X, they invert bits in the message: if $X = 1$, invert neither bit; if $X = 2$, invert 1st bit; if $X = 3$, invert 2nd bit; if $X = 4$, invert both bits.

Their idea is to send the value of X (the key to decoding the message) to the intended recipient by armored car. The driver of the car is mindful of traffic and weather and has been to evasive driving school.

You have been hired to consult with the engineers on the problem of how to pick X. The engineers wish to keep some information secret, such as the complete distribution for picking X. They also plan to give you some real information and some fake information. If you were supplied information in one (at a time) of the items below, could that information possibly come from a valid probability distribution? Answer yes or no for each item and justify your answer.

a) $P(1) = 0.2$, $P(2) = 0.3$, $P(1) - P(3) = 0.5$

b) $P(1) = 0$, $P(2) = 0$, $P(3) = 1$

c) $P(2) = 2/\pi$

d) $P(1) = 0.2$, $P(2) = 0.8$, $P(3) = 0.05$, $P(4) = 0.05$

e) $P(1 \text{ or } 2) = 0.2$, $P(2 \text{ or } 3) = 0.3$, $P(3 \text{ or } 4) = 0.5$

SOL'N: a) No. Given the value $P(1) = 0.2$, $P(3) = P(1) - 0.5 = -0.3 < 0$. Probability must be non-negative. Therefore, the information given cannot be part of a valid probability distribution.

b) Yes. We could have $P(4) = 0$, completing a valid probability distribution. Note that probability = 0 or 1 is possible and allowed so long as the total probability sums to one and all probabilities are non-negative and at most unity.

c) Yes. $0 < P(2) = 2/\pi \approx 2/3 < 1$ so this is a valid probability. We could easily assign probabilities for $P(1)$, $P(3)$, and $P(4)$ that would yield a valid probability distribution of which $P(2)$ is a part.

d) No. The sum of the probabilities exceeds 1.0 for all the outcomes. Since probabilities for all outcomes are listed, the sum of probabilities must equal unity.

e) No. If we consider just the probabilities of events "1 or 2" and "3 or 4", we must have a total probability = 1 since these two events account for all possible outcomes and do not overlap. (This is a total partition.) But P(1 or 2) + P(3 or 4) = 0.2 + 0.5 = 0.7 < 1.