EX: Consider a lie detector test. Notation for this problem is as follows:

$- \equiv$ detector says you Lied

$+ \equiv$ detector says you told the Truth

$L \equiv$ you did Lie

$T \equiv$ you told the Truth

The following information is given:

$P(-\,|\,L) = 0.89$

$P(+\,|\,L) = 0.11$

$P(-\,|\,T) = 0.1$

$P(+\,|\,T) = 0.9$

Determine the probability, $P(L|-)$, that you actually lied if the lie detector result says you lied. Our intuition might suggest an answer of approximately 90%.

SOL'N: Using Bayes' Theorem, we calculate the probability:

$$P(L\,|\,-) = \frac{P(-\,|\,L)P(L)}{P(-)}$$

where

$$P(-) = P(-\,|\,L)P(L) + P(-\,|\,T)P(T)$$

We need to know $P(L)$ and $P(T)$ to solve this problem.

Suppose we have the following additional information:

$$P(L) = 0.05$$

$$P(T) = 0.95$$

These values suggest that most people tell the truth. Using these values, we complete the calculation of the desired probability:

$$P(L\,|\,-) = \frac{0.89(0.05)}{0.89(0.05) + 0.1(0.95)} \approx 0.32$$

There is only a 32% chance you lied when the detector says you lied.