

EX: A company has manufactured ten power supplies marked with serial numbers 0 through 9.

- a) The company ships 3 power supplies to a customer in a box. These 3 power supplies are chosen at random from the original set of 10, (i.e., all of the 10 power supplies are equally likely to be chosen). Find the probability that the three power supplies in the box the customer receives are the power supplies with serial numbers 0, 2, and 5.
- b) Assume 4 of the 10 power supplies are faulty. The company ships 3 power supplies to a customer. These 3 power supplies are chosen at random from the original set of 10, (i.e., all of the 10 power supplies are equally likely to be chosen). Find the probability that the customer receives one or more faulty power supplies.

SOL'N: a) Since all sets of three power supplies are equally likely, the probability of choosing the one set consisting of 0, 2, and 5 is one divided by the number of ways of choosing three power supplies. Since the order in which the three power supplies does not matter, (only which supplies are chosen matters), the number of ways of choosing three power supplies is given by a combinatorial coefficient for picking 3 items out of 10.

$$P(\{0,2,5\}) = \frac{1}{10C_3} = \frac{1}{\frac{10 \cdot 9 \cdot 8}{3 \cdot 2 \cdot 1}} = \frac{1}{120}$$

- b) We simplify the calculation by calculating the probability for the complement of the event that the customer receives one or more bad power supplies. That is, we find the probability that all the power supplies are good and subtract this value from a probability of one:

$$P(1 \text{ or more bad}) = 1 - P(3 \text{ good})$$

Since all the power supplies are equally likely to be chosen, we have independence of the events of choosing a first power supply that is good, a second power supply that is good, and a third power supply that is good:

$$P(3 \text{ good}) = P(1\text{st good})P(2\text{nd good})P(3\text{rd good})$$

For the first power supply, we have 10 to choose from and 6 good supplies. Thus, we have 6 chances out of 10 of picking a good supply.

$$P(\text{1st good}) = 6/10$$

Since there are fewer power supplies to choose from after selecting the first one, we have sampling without replacement. If we succeeded at picking a good power supply the first time, we only have 9 good supplies left with 4 being bad. Thus, we have 5 chances out of 9 of picking a good supply.

$$P(\text{1st good}) = 5/9$$

For the third power supply, we have 8 good supplies left with 4 being bad. Thus, we have 4 chances out of 8 of picking a good supply.

$$P(\text{1st good}) = 4/8$$

Combining results gives our value for $P(3 \text{ good})$:

$$P(3 \text{ good}) = \frac{6 \cdot 5 \cdot 4}{10 \cdot 9 \cdot 8} = \frac{1}{6}$$

Our final answer is the complement of this value:

$$P(1 \text{ or more bad}) = 1 - P(3 \text{ good}) = \frac{5}{6}.$$

This result might seem surprising at first since more than half the power supplies were good. We might intuitively expect a probability closer to one-half, but it is unlikely that we will *not* pick *any* of the bad supplies in three tries.