EX: Consider the following expression:

$$P(\square \mid \square, \square) \cdot P(\square \mid C) \cdot P(C) = P(A, B, C)$$

In each of the terms with boxes, the letters A, B, and C appear at most once. (Thus, only an *A* or a *B* will be in the box in the second term.)

a) Determine the number of mathematically distinct equations we obtain by using all possible patterns of letters in the boxes. "Mathematically distinct" means we eliminate terms where the order of letters in boxes gives the same mathematical value as another order of those letters. An example of this idea would be P(A, B) = P(B, A).

b) Which of the distinct equations possible in part (a) *could* be true if the probabilities for *A*, *B*, and *C* are properly chosen? Justify your answers.

SOL'N: a) Where the comma separates events in the conditional part of the first term, the order of listing events doesn't matter: A, B is the same as B, A and so forth. Otherwise, all possible ways of listing letters will give different equations. In the second term, however, we only have A and B. Our list of expressions:

(i) $P(A \mid B, C) \cdot P(A \mid C) \cdot P(C) = P(A,B,C)$

(ii) $P(A \mid B, C) \cdot P(B \mid C) \cdot P(C) = P(A,B,C)$

(iii) $P(B \mid A, C) \cdot P(A \mid C) \cdot P(C) = P(A,B,C)$

(iv) $P(B \mid A, C) \cdot P(B \mid C) \cdot P(C) = P(A,B,C)$

(v) $P(C \mid A, B) \cdot P(A \mid C) \cdot P(C) = P(A,B,C)$

(vi) $P(C \mid A, B) \cdot P(B \mid C) \cdot P(C) = P(A,B,C)$

b) First, we observe that (ii) and (iii) *must* be true. In (ii), for example, we have $P(B \mid C) \cdot P(C) = P(B, C)$ and we have the standard conditional probability equation of the form $P(A \mid D) \cdot P(D) = P(A, D)$ where $D \equiv (A, B)$.

Second, we consider the possibility that A and B are the same event. Then (i) and (iv) are the same as (ii) and (iii). Thus, (i) and (iv) *could* be true.

In a similar vein, (v) and (vi) could be true if A, B, and C are all the same event. In that case, the conditional probabilities are all equal to one, and $P(A, B, C) = P(A) = P(B) = P(C)$.

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