EX: Find the probability density function of the average value of 12 independent standard gaussian random variables.

SOL'N: We want to find the probability density function for Y:

$$Y = \frac{1}{12}\sum_{i=1}^{12} X_i$$

where

$X_i \sim \mathrm{n}(0,1)$ are independent standard gaussian random variables

We rewrite the expression for Y to emphasize that it is a linear combination of independent random variables.

$$Y = \frac{1}{12}X_1 + \ldots + \frac{1}{12}X_{12}$$

Thus, we have the following mean and variance:

$$\mu_Y = \frac{1}{12}\mu_{X_1} + \ldots + \frac{1}{12}\mu_{X_{12}} = 12 \cdot \frac{1}{12} \cdot 0 = 0$$

$$\sigma_Y^2 = \left(\frac{1}{12}\right)^2 \sigma_{X_1}^2 + \ldots + \left(\frac{1}{12}\right)^2 \sigma_{X_{12}}^2 = 12 \cdot \left(\frac{1}{12}\right)^2 \cdot 1 = \frac{1}{12}$$

Also, the probability density function (pdf) for a linear combination of independent gaussian random variables is a gaussian random variable. Thus, the pdf for Y is the following gaussian distribution:

$$f_Y(y) = \frac{1}{\sqrt{2\pi\sigma_Y^2}} e^{-(y-\mu_Y)^2/2\sigma_Y^2}$$

NOTE: The pdf for a linear combination of independent gaussian random variables, X_i, is gaussian even when the X_i have differing means and variances.