EX: Given X~u(0, 1), (i.e., X is uniformly distributed from 0 to 1), and $Y = 5X + 1$, find the following values:

a) μ_Y.

b) σ_{XY}.

SOL'N: a) The mean of Y is given by a standard formula:

$$\mu_{Y=aX+b} = a\mu_X + b$$

Substituting values given in the problem, we have the following result:

$$\mu_{Y=5X+1} = 5\mu_X + 1$$

The mean value of X is 1/2 since it has a uniform distribution on (0, 1). Making this substitution gives our answer:

$$\mu_Y = 5\frac{1}{2} + 1 = \frac{7}{2}$$

b) The covariance, σ_{XY}, is defined in terms of $E(XY)$:

$$\sigma_{XY} = E(XY) - \mu_X\mu_Y$$

If we substitute $Y = 5X + 1$, we are left with only expected values involving x:

$$\sigma_{XY} = E(X(5X+1)) - \mu_X\mu_{5X+1} = E(5X^2 + X) - \mu_X\mu_{5X+1}$$

Using the result from (a), the second term simplifies as follows:

$$\mu_X\mu_{5X+1} = \frac{1}{2}\cdot\frac{7}{2} = \frac{7}{4}$$

The first term becomes a sum:

$$E(5X^2 + X) = 5E(X^2) + E(X) = 5E(X^2) + \frac{1}{2}$$

To find the expected value of X^2, we consider the variance of X. For a uniform distribution on (0, 1), the variance is 1/12.

$$\sigma_X^2 = \frac{1}{12} = E(X^2) - \mu_X^2 = E(X^2) - \left(\frac{1}{2}\right)^2$$

Solving for $E(X^2)$, we have the following:

$$E(X^2) = \sigma_X^2 + \mu_X^2 = \frac{1}{12} + \frac{1}{4} = \frac{1}{3}$$

Substituting this result in our earlier equation, we have the information needed to complete the calculation of covariance:

$$E(5X^2 + X) = 5E(X^2) + E(X) = 5 \cdot \frac{1}{3} + \frac{1}{2}$$

and

$$\sigma_{XY} = E(X(5X+1)) - \mu_X \mu_{5X+1} = \frac{13}{6} - \frac{7}{4} = \frac{26-21}{12} = \frac{5}{12}$$