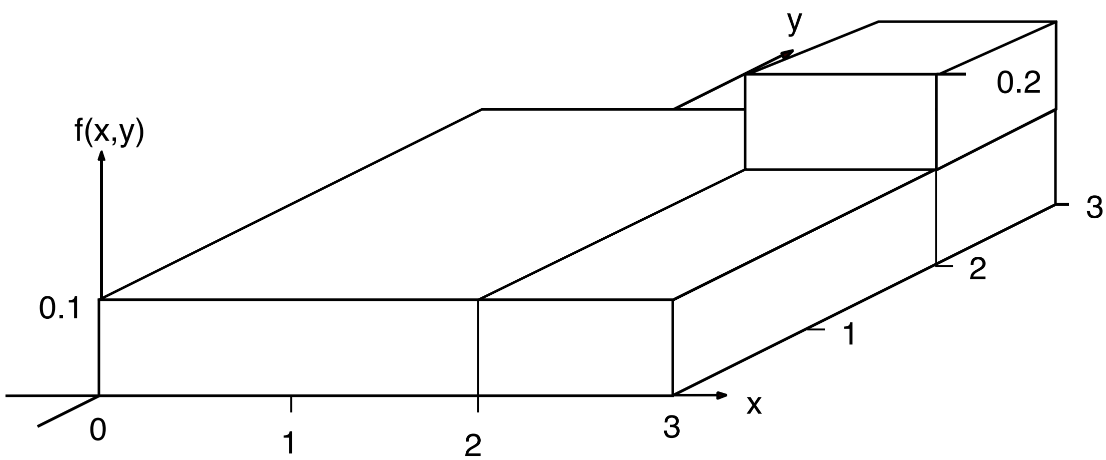
EX: Find μ_X for the joint probability function shown below.



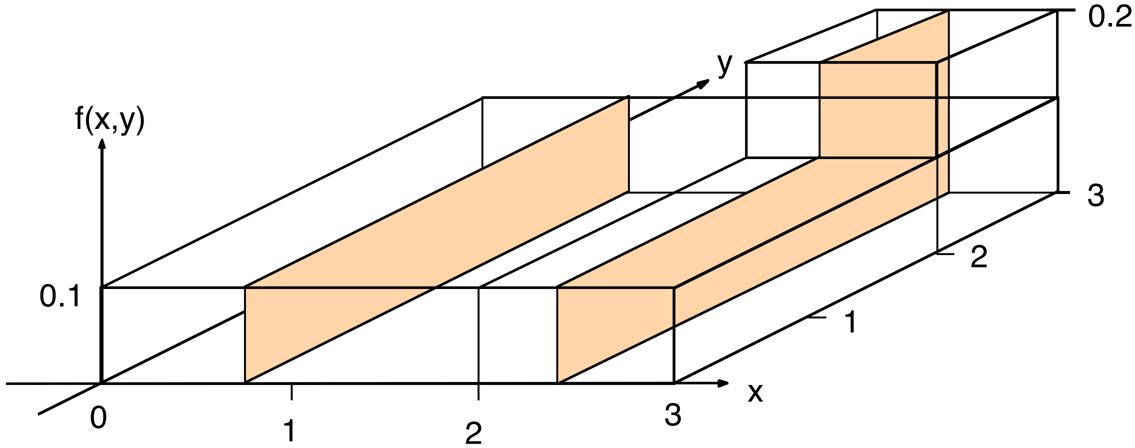
SOL'N: We may express $f(x, y)$ as a sum of two boxes:

$$f(x,y) = \begin{cases} 0.1 & 0 \le x \le 3 \text{ and } 0 \le y \le 3 \\ 0 & \text{otherwise} \end{cases} + \begin{cases} 0.1 & 2 \le x \le 3 \text{ and } 2 \le y \le 3 \\ 0 & \text{otherwise} \end{cases}$$

We integrate in the y direction to find the marginal density function, $f_X(x)$, that we need in the calculation of μ_X.

$$f_X(x) = \int_{-\infty}^{\infty} yf(x,y)dx = \begin{cases} \int_{y=0}^{3} y(0.1)dy & 0 \le x \le 2 \\ \int_{y=0}^{3} y(0.1)dy + \int_{y=2}^{3} y(0.1)dy & 2 \le x \le 3 \\ 0 & \text{otherwise} \end{cases}$$

We can calculate the above integrals, or we may observe that the values of the integrals represent the areas of cross sections in the y direction, as indicated in the following figure.



The areas are given by widths times heights.

$$f_X(x) = \begin{cases} 0.3 & 0 \le x \le 2 \\ 0.3 + 0.1 = 0.4 & 2 \le x \le 3 \\ 0 & \text{otherwise} \end{cases}.$$

Now we integrate over x to find $\mu_X \equiv E(X)$.

$$\mu_X \equiv E(X) = \int_{-\infty}^{\infty} x f_X(x)\,dx = \int_0^2 x(0.3)\,dx + \int_2^3 x(0.4)\,dx$$

or

$$\mu_X = 0.3\frac{x^2}{2}\bigg|_{x=0}^{x=2} + 0.4\frac{x^2}{2}\bigg|_{x=2}^{x=3}$$

or

$$\mu_X = 0.3 \cdot \frac{4}{2} + 0.4 \cdot \left(\frac{9}{2} - \frac{4}{2}\right) = 1.6$$