EX: An engineer is analyzing a circuit in which there is a diode. The current in the diode is given by the following equation:

$$i_D = I_0(e^{v/V_T} - 1)$$

If $Y \equiv v/V_T$ is gaussian distributed with mean value $\mu_Y = 0.7\text{V}/26\text{mV}$ and variance $\sigma_Y^2 = 0.02\text{ V}$, find the probability density function of $Z = \dfrac{i_D}{I_0}$.

SOL'N: We observe that, given Y is gaussian distributed, the following random variable, X, has a lognormal distribution:

$$X = e^Y$$

This is slightly different from the Z we are interested in, but it differs only by a horizontal shift by a value of one:

$$Z = X - 1$$

Thus, we start with the probability density function for Y and then determine how to make the shift to Z.

$$f_X(x) = \begin{cases} \dfrac{1}{x\sqrt{2\pi\sigma_Y^2}} e^{-[\ln(x)-\mu_Y]^2/2\sigma_Y^2} & x > 0 \\ 0 & \text{otherwise} \end{cases}$$

Now we write X in terms of Z:

$$X = Z + 1$$

Making this substitution for X we obtain the following result:

$$f_Z(z) = f_X(X = Z + 1)$$

or

$$f_Z(z) = \begin{cases} \dfrac{1}{(z+1)\sqrt{2\pi\sigma_Y^2}} e^{-[\ln(z+1)-\mu_Y]^2/2\sigma_Y^2} & z + 1 > 0 \\ 0 & \text{otherwise} \end{cases}$$

or

$$f_Z(z) = \begin{cases} \dfrac{1}{(z+1)\sqrt{2\pi\sigma_Y^2}} e^{-[\ln(z+1)-\mu_Y]^2/2\sigma_Y^2} & z > -1 \\ 0 & \text{otherwise} \end{cases}$$

NOTE: To determine whether we should add or subtract one when substituting for x, we observe that when $x = 0$ we have $z = -1$, and we should have the same probability density for $x = 0$ and $z = -1$. This implies that a term equal to x in $f_X(x)$ should be replaced by a term that adds 1 to z so the value of the term will again be zero.