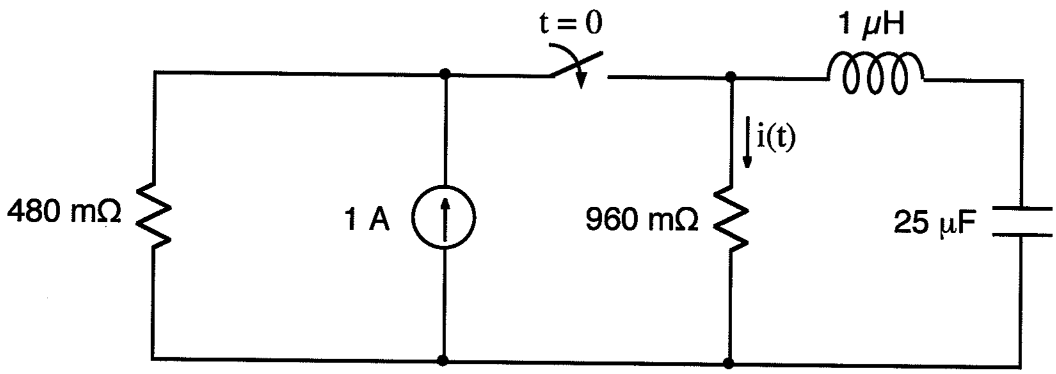
Ex:



After being open for a long time, the switch closes at $t = 0$.

Find $i(t)$ for $t > 0$.

sol'n: Find characteristic roots using circuit for $t>0$. Set source to zero find R_{Thev} for roots:

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2} \quad , \quad \alpha = \frac{R}{2L} \quad , \quad \omega_0^2 = \frac{1}{LC}$$

$$R_{Thev} = 960\,m\Omega \,\|\, 480\,m\Omega = 480\,m\Omega \cdot 2\|1 = 320\,m\Omega$$

$$\alpha = \frac{320\,m\Omega}{2 \cdot 1\,\mu H} = 160\,k\ r/s \qquad \omega_0^2 = \frac{1}{1\mu H \cdot 25\mu F}$$

$$\alpha^2 - \omega_0^2 < 0 \text{ so underdamped} \qquad \omega_0^2 = \left(\frac{1}{5}M\,r/s\right)^2$$

or

$$\omega_d = \sqrt{\omega_0^2 - \alpha^2} = \sqrt{(200k)^2 - (160k)^2}\ r/s, \quad \omega_0^2 = (120k\ r/s)^2$$

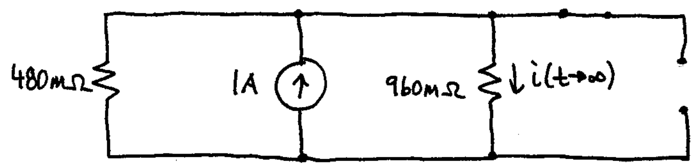
$$\omega_d = 120\,k\ r/s$$

$$s_{1,2} = -160k \pm 120\,k\ r/s = -40k\ r/s \text{ and } -280k\ r/s$$

Use general soln $i(t) = A_1 e^{-\alpha t}\cos(\omega_d t) + A_2 e^{-\alpha t}\sin(\omega_d t) + A_3$.

$A_3 =$ final value

For $t \to \infty$, L = wire, C = open, switch closed.



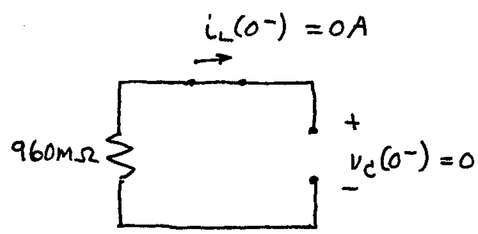
We have a current divider.

$$i(t \to \infty) = 1A \cdot \frac{480\,m\Omega}{480\,m\Omega + 960\,m\Omega} = \frac{1}{3}\,A$$

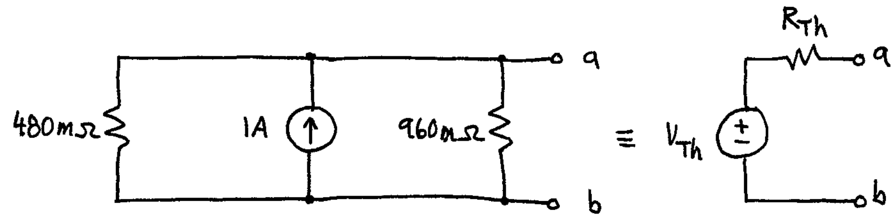
Now find $i(0^+)$ and $\left.\frac{di(t)}{dt}\right|_{t=0^+}$.

Start at $t = 0^-$ and find $i_L(0^-)$, $v_C(0^-)$.
(Then we'll use $i_L(0^+) = i_L(0^-)$,
$v_C(0^+) = v_C(0^-)$.)

At $t = 0^-$, L = wire, C = open, switch open.



For $t = 0^+$, one approach is to take a Thevenin equivalent of the durrent source and R's.



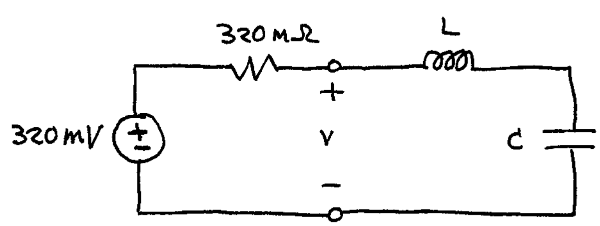
$V_{Th} = v_{a,b}$ with nothing attached to a,b.

$V_{Th} = 1A \cdot 480m\Omega \| 960m\Omega = 1A \cdot 320m\Omega = 320mV$

R_{Th} = resistance seen looking into a,b with 1A source turned off

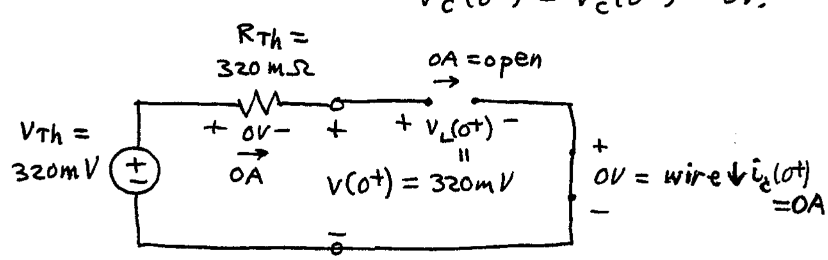
$R_{Th} = 480\,m\Omega \| 960m\Omega = 320\,m\Omega$ (as noted above)

We now find $v(t)$ in our new circuit and use $i(t) = v(t) / 960m\Omega$ from Ohm's law.



At $t=0^+$ we have $i_L(0^+) = i_L(0^-) = 0A$

$v_C(0^+) = v_C(0^-) = 0V.$



$v(0^+) = 320\,mV$ from above circuit

We match this to symbolic $v(0^+)$:

$$v(t) = A_1 e^{-\alpha t}\cos(\omega_d t) + A_2 e^{-\alpha t}\sin(\omega_d t) + A_3$$

$$v(0^+) = A_1 + A_3$$

What is A_3 for $v(t)$? It will be the A_3 we found for $i(t)$ multiplied by $960\,m\Omega$, (by Ohm's Law).

$$A_3 = \tfrac{1}{3}A \cdot 960\,m\Omega = 320\,mV \quad (= 1A \cdot R_{Th})$$

Back to $v(0^+)$, we have

$$v(0^+) = A_1 + \underbrace{A_3}_{=320mV} = 320\,mV \text{ from circuit}$$

$\therefore A_1 = 0$

Now we find $\left.\dfrac{dv(t)}{dt}\right|_{t=0^+}$ by writing

$v(t)$ in terms of state vars i_L and v_C.

We must <u>not</u> plug in values until after we take d/dt.

$v(t) = V_{Th} - R_{Th}\, i_L$ works since i_L is state var

$$\frac{dv(t)}{dt} = \underbrace{\frac{d\,\cancel{V_{Th}}}{dt}}_{0 \text{ since } V_{Th} = \text{const}} - R_{Th}\frac{di_L}{dt}$$

Now use $\dfrac{di_L}{dt} = \dfrac{v_L}{L}$, $\left(\text{and } \dfrac{dv_C}{dt} = \dfrac{i_C}{C} \text{ usually}\right)$.

$$\frac{dv(t)}{dt} = -R_{Th}\frac{v_L}{L}$$

$$\left.\frac{dv(t)}{dt}\right|_{t=0^+} = -\frac{R_{Th}}{L} v_L(0^+) = -\frac{320\,m\Omega}{1\mu H}\cdot 320\,mV$$

From symbolic $v(t)$ we have

$$\left.\frac{dv(t)}{dt}\right|_{t=0^+} = A_1(-\alpha) + A_2\omega_d$$

Thus, $A_1(-\alpha) + A_2\omega_d = -\frac{320}{1\mu H}m\Omega \cdot 320\,mV$

But $A_1 = 0$. $\therefore A_2 = -\frac{320\,m\Omega \cdot 320\,mV}{1\mu H \cdot (\omega_d = 120k\,r/s)}$

$$v(t) = -\frac{320\,m\Omega \cdot 320\,mV}{1\mu H \cdot 120k\,r/s} e^{-160kt} \sin(120kt) + 320\,mV$$

$$i(t) = \frac{v(t)}{960\,m\Omega} \quad \text{since } v \text{ is across } 960\,m\Omega$$

$$i(t) = -\frac{320\,\cancel{\mu}V \cdot 1/3}{1\cancel{\mu}H \cdot 120\cancel{k}\,r/s} e^{-160kt} \sin(120kt) + \frac{1}{3}A$$

$$i(t) = -\frac{8A}{9}e^{-160kt} \sin(120kt) + \frac{1}{3}A$$