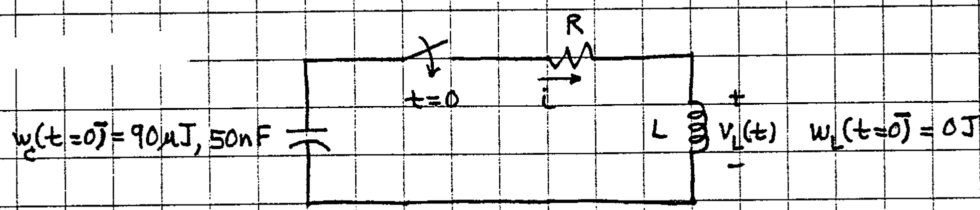


roots of char eq'n for $i$ are $s_1 = -1000\ s^{-1}$  $s_2 = -4000\ s^{-1}$

a) Find $R$ and $L$

b) Find $i(t=0^+)$ and $\left.\dfrac{di(t)}{dt}\right|_{t=0^+}$

c) Find $i(t \geq 0)$

d) Find $t$ when $i(t)$ is max

e) Find max $i(t)$

f) Find $v_L(t \geq 0)$

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ans: a) $R = 25\,k$, $L = 5H$

b) $i(0^+) = 0$, $\left.\dfrac{di(t)}{dt}\right|_{t=0^+} = 12\,A/s$

c) $i(t \geq 0) = 4e^{-1kt} - 4e^{-4kt}$ mA

d) $t = 462.10\,\mu s$

e) $i_{max} = 1.89$ mA

f) $v_L(t \geq 0) = -20e^{-1kt} + 80e^{-4kt}$ V

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soln: a) $s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2}$  $s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2}$

$$s_1 + s_2 = -2\alpha = \frac{-2R}{2L} = -\frac{R}{L} \quad \text{or} \quad R = -L(s_1 + s_2) = 2\alpha L$$

$$\alpha = \frac{s_1 + s_2}{-2} = \frac{-1k - 4k}{-2}\ s^{-1} = 2.5\ k/s$$

$$s_1 - s_2 = 2\sqrt{\alpha^2 - \omega_0^2}$$

$$\| \qquad\qquad \|$$

$$-1k - -4k\ s^{-1} = 2\sqrt{(2.5k)^2 - \omega_0^2}$$

$$\| \qquad\qquad \|$$

$$3k/s \quad = \qquad\qquad$$ *Now square both sides.*

$$(3k)^2 = 4\cdot\left[(2.5k)^2 - \omega_0^2\right]$$

$$(1.5k)^2 = (2.5k)^2 - \omega_0^2$$

$$\omega_0^2 = (2.5k)^2 - (1.5k)^2 = (0.5k)^2(5^2 - 3^2) = (0.5k)^2 \cdot 4^2$$

$$\omega_0 = 2k/s$$

$$\omega_0 = \frac{1}{\sqrt{LC}}, \quad \omega_0^2 = \frac{1}{LC}, \quad L = \frac{1}{\omega_0^2 C}$$

$$L = \frac{1}{4M \cdot 50n} H = \frac{1k}{200} = 5 H$$

$$R = 2\alpha L = 2 \cdot 2.5k \cdot 5\ \Omega = 25k\Omega$$

b) $i(t=0^+) = i(t=0^-)$ since $i$ in $L$ cannot change instantly
$= 0\ A$ since energy stored in $L$ is $w_L = \frac{1}{2} L i^2 = 0 J$
$\therefore i(t=0^-) = 0$

For $\left.\frac{di(t)}{dt}\right|_{t=0^+}$ we use $v_L(t=0^+) = L \left.\frac{di(t)}{dt}\right|_{t=0^+}$

But since $i(t=0^+) = 0A$, there is no V-drop across R,
and $v_L(t=0^+) = v_C(t=0^+)$.

$v_C(t=0^+) = v_C(t=0^-)$ since $v_C$ cannot change instantly.

Energy in C is $w_C(t=0^-) = \frac{1}{2} C v_C^2(t=0^-) = 90\mu J$

$$\therefore v_C(t=0^-) = \sqrt{\frac{2 \cdot 90\mu J}{50nF}} = \sqrt{\frac{90k}{25}} = \frac{300}{5} = 60V$$

So $v_L(t=0^+) = 60V = 5H \cdot \left.\frac{di(t)}{dt}\right|_{t=0^+}$

$$\therefore \left.\frac{di(t)}{dt}\right|_{t=0^+} = \frac{60}{5} A/s = 12\ A/s$$

c) $i(t \geq 0) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$

Match initial conditions $i(t=0^+)$ and $\left.\frac{di(t)}{dt}\right|_{t=0^+}$.

$$i(t=0^+) = A_1 e^{s_1 0} + A_2 e^{s_2 0} = A_1 + A_2 = 0\ A$$

$$\left.\frac{di(t)}{dt}\right|_{t=0^+} = s_1 A_1 e^{s_1 \cdot 0} + s_2 A_2 e^{s_2 \cdot 0} = s_1 A_1 + s_2 A_2 = 12\ A/s$$

$$A_2 = -A_1$$

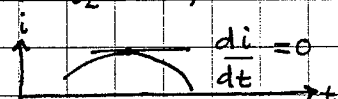
$$s_1 A_1 + s_2 A_2 = s_1 A_1 - s_2 A_1 = (s_1 - s_2) A_1 = 12 \text{ A/s}$$

$$s_1 - s_2 = -1\text{k} - -4\text{k} = 3\text{k/s}$$

$$A_1 = \frac{12}{3\text{k}} = 4 \text{ mA} \qquad A_2 = -4 \text{ mA}$$

$$\boxed{i_L(t \geq 0) = 4e^{-1\text{k}t} - 4e^{-4\text{k}t} \text{ mA}}$$

d) max $i_L(t \geq 0)$ occurs when $\dfrac{di_L(t)}{dt} = 0$ A/s.




$\dfrac{di}{dt} = 0$ at max

$$\frac{di_L(t)}{dt} = -4\text{k}\, e^{-1\text{k}t} + 16\text{k}\, e^{-4\text{k}t} = 0$$

$$16\text{k}\, e^{-4\text{k}t} = 4\text{k}\, e^{-1\text{k}t}$$

$$\frac{e^{-4\text{k}t}}{e^{-1\text{k}t}} = \frac{1}{4}$$

$$e^{(-4\text{k} - -1\text{k})t} = \frac{1}{4}$$

$$e^{-3\text{k}t} = \frac{1}{4}$$

$$-3\text{k}t = \ln\frac{1}{4} = -\ln 4$$

$$t = \frac{\ln 4}{3\text{k}} = 462.1\ \mu\text{s}$$

e) $$i(t)_{max} = i_L(t = 462.1\ \mu\text{s}) = 4e^{-1\text{k}\ln 4/3\text{k}} - 4e^{-4\text{k}\ln 4/3\text{k}} \text{ mA}$$

$$\parallel\!\parallel$$

$$i_{max} = 4 \cdot 4^{-1/3} - 4 \cdot 4^{-4/3} \text{ mA} = (4 - 1)4^{-1/3} \text{ mA} = \frac{3}{4^{1/3}} = 1.89 \text{ mA}$$