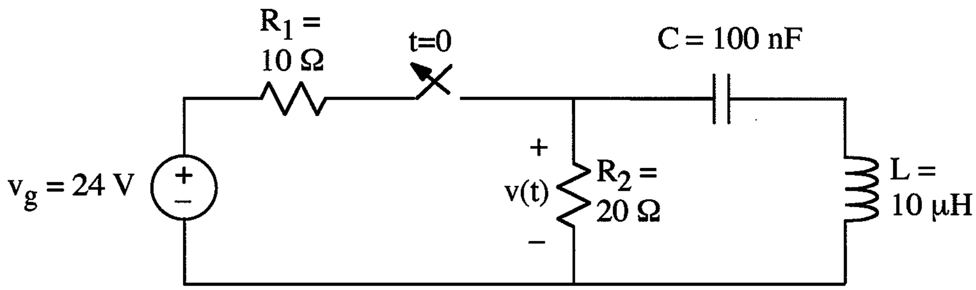
Ex:



After being closed for a long time, the switch opens at t = 0.

a) State whether v(t) is underdamped, overdamped, or critically damped.

b) Write a numerical time-domain expression for v(t), t > 0, the voltage across R_2. This expression must not contain any complex numbers.

sol'n: a) Characteristic roots from series R_2LC circuit after $t = 0^+$.

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}, \quad \alpha = \frac{R_2}{2L}, \quad \omega_0^2 = \frac{1}{LC}$$

$$\alpha = \frac{20\,\Omega}{2 \cdot 10\mu H} = 1\text{M r/s}$$

$$\omega_0^2 = \frac{1}{10\mu H \cdot 100nF} = \frac{1}{1\mu^2} \text{ r}^2/\text{s}^2 = (1\text{M r/s})^2$$

$$\sqrt{\alpha^2 - \omega_0^2} = \sqrt{(1\text{M r/s})^2 - (1\text{M r/s})^2} = 0$$

∴ The circuit is critically damped.

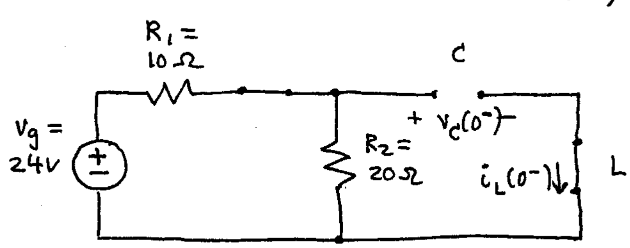
$$s = -\alpha = -1\text{M r/s}$$

b) Our soln is of form

$$v(t) = A_1 e^{st} + A_2 t e^{st} + (A_3 = 0) \text{ (no src as } t \to \infty)$$

Now find $v(0^+)$ and $\left.\frac{dv(t)}{dt}\right|_{t=0^+}$ from circuit.

Start at $t=0^-$ when L = wire, C = open.



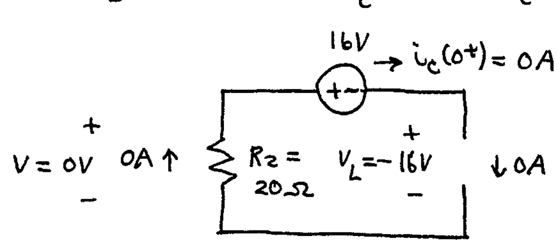
Find $i_L(0^-)$ and $v_C(0^-)$, (energy vars).

$i_L(0^-) = 0$ since C = open

$v_C(0^-) = v_{R_2} = \dfrac{v_g R_2}{R_1 + R_2}$ v-divider

$v_C(0^-) = 24V \cdot \dfrac{20}{10+20} = 16V$

Now solve circuit at $t=0^+$ with
$i_L(0^+) = i_L(0^-)$ and $v_C(0^+) = v_C(0^-)$:



We see that $v(0^+) = 0V$.

$v(0^+) = A_1$ from symbolic form.

$\therefore\ A_1 = 0\text{ V}$ matching circuit value to symbolic form

Now find $\left.\dfrac{dv(t)}{dt}\right|_{t=0^+}$ from circuit.

We write $v(t)$ in terms of i_L and v_C.
(Write an eq'n that is true for all $t>0$.
Plug in specific values only <u>after</u> taking derivative.)

$v(t) = -i_L R_2$ since i_L flows thru R_2

$$\frac{dv(t)}{dt} = -R\frac{di_L}{dt} = -R\frac{v_L}{L} \quad \text{from } v_L = L\frac{di_L}{dt}$$

$$\left.\frac{dv(t)}{dt}\right|_{t=0^+} = -R\frac{v_L(0^+)}{L} = \frac{20\Omega\cdot 16\text{V}}{10\mu\text{H}} \quad (\text{see circuit for } t=0^+)$$

$$\left.\frac{dv(t)}{dt}\right|_{t=0^+} = 32\ \text{MV/s}$$

From symbolic form

$$\left.\frac{dv(t)}{dt}\right|_{t=0^+} = A_1 s\, e^{0^+} + A_2 e^{0^+} + \overset{0}{\cancel{A_2\cdot 0^+ s_2 e^{0^+}}}$$

$$\left.\frac{dv(t)}{dt}\right|_{t=0^+} = A_1 s + A_2$$

Equate symbolic and circuit values.

$$A_1 s + A_2 = 32 \text{ MV/s}$$

From before, $A_1 = 0$.

Thus, $A_2 = 32 \text{ MV/s}$

$$v(t) = 32 \text{ M}\frac{\text{V}}{\text{s}} \cdot t \cdot e^{-1\text{M}t}$$