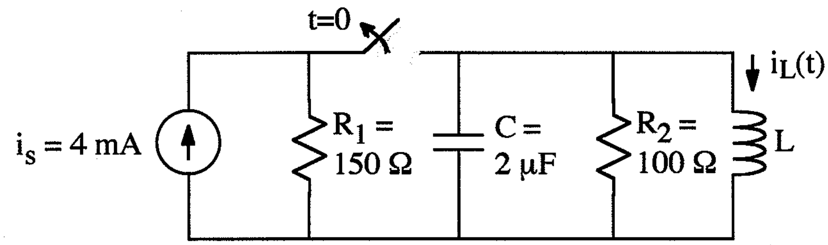
EX:



After being closed for a long time, the switch opens at t = 0.

a) If L = 125 mH, find the characteristic roots, s_1 and s_2, for the above circuit.

b) If L = 11.834 mH, find the damping frequency, ω_d.

c) Find the value of L that makes the circuit critically damped.

SOL'N: a) After t=0 we have parallel RLC.

From the differential eq'n for the parallel RLC with Ae^{st} substituted for the sol'n, we get the characteristic eq'n for the circuit:

$$s^2 + \frac{1}{RC}s + \frac{1}{LC} = 0$$

We get the characteristic roots by solving this quadratic eq'n.

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$$

where $\alpha \equiv \frac{1}{2RC}$ $\omega_0^2 \equiv \frac{1}{LC}$

Now we plug in values for components.

$R = R_2 = 100\,\Omega$ $L = 125$ mH (for part a) $C = 2\mu F$

$$\alpha = \frac{1}{2 \cdot 100\Omega \, 2\mu F} = \frac{1}{400} \text{ M rad/s} = \frac{2.5}{1k} \text{ M rad/s}$$

$$\alpha = 2.5 \text{ k rad/s}$$

$$\omega_o^2 = \frac{1}{125 \text{ mH } 2\mu F} = \frac{1}{250 \text{ n}} (\text{rad/s})^2 = \frac{4}{1k} \text{ G } (\text{rad/s})^2$$

$$\omega_o^2 = 4 \text{ M} (\text{rad/s})^2 \quad \text{or} \quad \omega_o = 2k \text{ rad/s}$$

$$\therefore \quad s_{1,2} = -2.5k \pm \sqrt{(2.5k)^2 - (2k)^2} \quad \text{rad/s}$$

$$'' = -2.5k \pm 1.5k \quad \text{rad/s}$$

$$s_{1,2} = -4k \text{ and } -1k \quad \text{rad/s}$$

Note: Real part of s is always ≤ 0 for an RLC circuit.

b) For $L = 11.834$ mH we have different ω_o^2:

$$\omega_o^2 = \frac{1}{LC} = \frac{1}{11.834 \text{ mH } 2\mu F} \doteq \frac{42.25 \text{ m}}{\text{ns}} \text{ rad}$$

$$\omega_o^2 = 42.25 \text{ M rad/s} \quad \text{or} \quad \omega_o = 6.5k \text{ rad/s}$$

Note: Changing L does not change α for a parallel RLC, but it would change α for a series RLC where $\alpha = R/2L$.

Here, $s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_o^2}$ with $\omega_o > \alpha$.

Since $\omega_o > \alpha$, we get complex roots:

$$s_{1,2} = -2.5\,k \pm \sqrt{(2.5k)^2 - (6.5k)^2} \quad rad/s$$

$$s_{1,2} = -2.5\,k \pm j\,6.0\,k \quad rad/s$$

We define the damping frequency to be the magnitude of the $\sqrt{\;}$ term, or

$$\omega_d = \sqrt{\omega_o^2 - \alpha^2} \quad \text{(order of } \omega_o^2, \alpha^2 \text{ is reversed)}$$

$$\omega_d = \sqrt{(6.5k)^2 - (2.5k)^2} \quad rad/s$$

$$\omega_d = 6k \quad rad/s$$

c) The circuit is critically damped when $s_1 = s_2$, i.e. when $\sqrt{\;} = 0$ or $\alpha = \omega_o$.

Since $\alpha = 2.5\,k$ rad/s doesn't change with L,

we must have $\omega_o = 2.5k$ rad/s.

$$\therefore \frac{1}{\sqrt{LC}} = 2.5k \text{ rad/s} \Rightarrow \frac{1}{LC} = (2.5\,k)^2 (r/s)^2$$

$$\text{or} \quad L = \frac{1 \quad (s/r)^2}{(2.5k)^2\,C} = \frac{1 \quad H}{6.25\cancel{k^2}\,2\cancel{\mu}} = \frac{1\,H}{6.25 \cdot 2}$$

$$L = 80 \text{ mH}$$