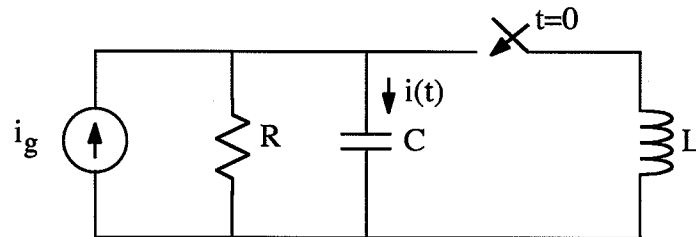


EX:



After being open for a long time, the switch closes at  $t = 0$ .

The inductor carries no current at time  $t = 0^-$ .

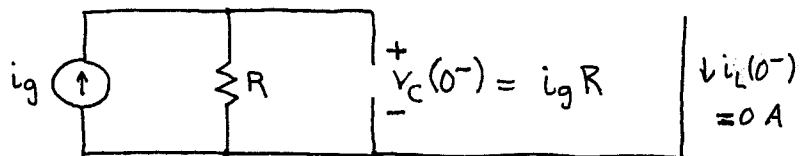
- a) Give expressions for the following in terms of  $i_g$ ,  $R$ ,  $L$ , and  $C$ :

$$i(t = 0^+) \quad \text{and} \quad \left. \frac{di(t)}{dt} \right|_{t=0^+}$$

- b) Find the numerical values of  $L$  and  $R$  given the following information:

$$C = 5 \mu\text{F} \quad s_1 = -10\text{k rad/s} \quad s_2 = -40\text{k rad/s}$$

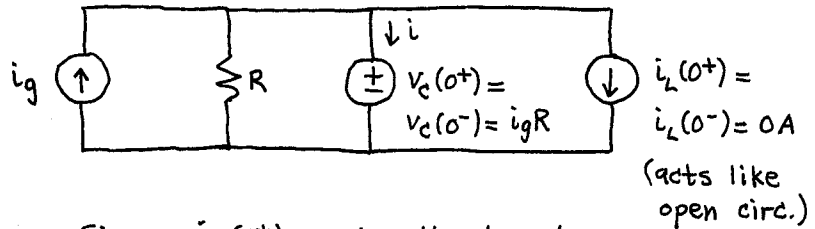
sol'n: a) We find initial conditions by starting at  $t = 0^-$ : ( $L$  acts like wire,  $C$  acts like open)



$$i_L(0^-) = 0\text{ A} \quad \text{since prob says so}$$

We only find the values of these energy variables at  $t = 0^-$  because they will not change instantly when we close the switch.

At  $t=0^+$ : (model L as i src, C as v src)



Since  $i_L(0^+) = 0A$ , the L acts like it is not there still.

The current thru R is  $v_c(0^+)/R = i_g$ .  
Thus, no current is left to flow thru C, and  $i(0^+) = 0A$ .

To find  $\left. \frac{di}{dt} \right|_{t=0^+}$ , we start by

writing  $i$  in terms of  $i_L$  and  $v_c$ .  
(Don't plug in  $t=0^+$  or take  $d/dt$  yet.)

Summing current out of the top wire:

$$-i_g + i_R + i + i_L = 0A$$

Replacing  $i_R$  with  $v_c/R$ , we have

$$i = i_g - \left( \frac{v_c}{R} + i_L \right).$$

Taking  $\frac{d}{dt}$  of both sides:

$$\frac{di}{dt} = -\frac{1}{R} \frac{dv_c}{dt} - \frac{di_L}{dt}$$

We use  $\frac{dv_c}{dt} = \frac{i_c}{C}$  and  $\frac{di_L}{dt} = \frac{v_L}{L}$ :

$$\frac{di}{dt} = -\frac{1}{R} \frac{v_c}{C} - \frac{v_L}{L}$$

$$\therefore \left. \frac{di}{dt} \right|_{t=0^+} = -\frac{1}{RC} i_c(0^+) - \frac{v_L(0^+)}{L}$$

Returning to our circuit for  $t=0^+$ ,  
we have  $i_c(0^+) = i(0^+) = 0A$ .

$$\text{Also, } v_L(0^+) = v_c(0^+) = i_g R.$$

$$\left. \frac{di}{dt} \right|_{t=0^+} = -\frac{i_g R}{L}$$

b) We always have  $s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$ .

We have a parallel RLC with  $\alpha = \frac{1}{2RC}$ .

For any simple RLC,  $\omega_0^2 = \frac{1}{LC}$ .

From the above, we use

$$s_1 + s_2 = -2\alpha = -\frac{1}{2RC} \cdot 2$$

$$\text{or } -10 \text{ k r/s} - 40 \text{ k r/s} = -\frac{1}{R \cdot 5 \mu\text{F}}$$

$$\text{or } R = \frac{-1 \text{ } \Omega}{-50 \text{ k} \cdot 5 \mu} = \frac{1 \text{ } \Omega}{250 \text{ m}} = 4 \Omega$$

$$R = 4 \Omega$$

(and  $\alpha = 25 \text{ k/s}$ )

To find  $L$ , we use

$$\begin{aligned} s_1 \cdot s_2 &= \left(-\alpha + \sqrt{\alpha^2 - \omega_0^2}\right) \left(-\alpha - \sqrt{\alpha^2 - \omega_0^2}\right) \\ &= (-\alpha)^2 - \sqrt{\alpha^2 - \omega_0^2}^2 \\ &= \alpha^2 - (\alpha^2 - \omega_0^2) \\ &= \omega_0^2 \\ &= \frac{1}{LC} \end{aligned}$$

$$\therefore L = \frac{1}{s_1 s_2 C}$$

$$L = \frac{1}{(-10\text{k})(-40\text{k})5\mu} \text{ H}$$

$$L = \frac{1}{2000} \text{ H}$$

$$L = 500 \mu\text{H}$$