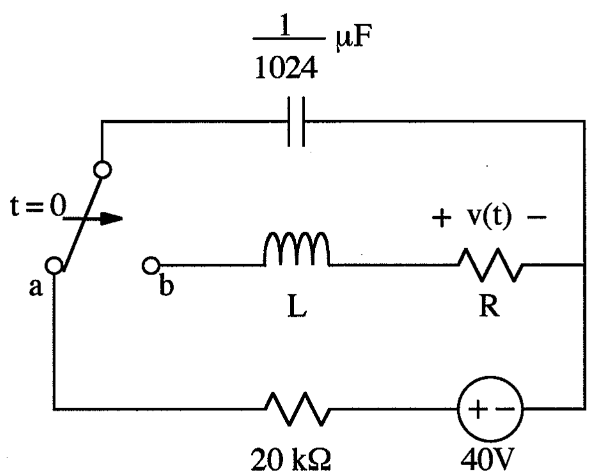
Ex:



In the above circuit, the switch moves from a to b at t = 0.

Find values of R and L such that the roots of the characteristic equation of the circuit for t > 0 are $s_1 = -8$ k rad/s and $s_2 = -512$ k rad/s.

Sol'n: After time t = 0, the circuit is a series RLC consisting of R, L, and C.

$$\therefore \ \alpha = \frac{R}{2L}$$

The characteristic roots are

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$$

where

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

The product of characteristic roots is ω_0^2:

$$s_1 \cdot s_2 = \left(-\alpha + \sqrt{\alpha^2 - \omega_0^2}\right)\left(-\alpha - \sqrt{\alpha^2 - \omega_0^2}\right)$$

$$s_1 \cdot s_2 = (-\alpha)^2 - \sqrt{\alpha^2 - \omega_0^2}^{\,2}$$

$$'' \quad = \alpha^2 - (\alpha^2 - \omega_0^2)$$

$$s_1 \cdot s_2 = \omega_0^2$$

Using $\omega_0^2 = \frac{1}{LC}$, we have

$$L = \frac{1}{C\, s_1 s_2} = \frac{1}{\frac{1}{1024}\mu F \cdot 8k \cdot 512k}$$

$$= \frac{1024}{4096} \text{ H}$$

$$L = 250 \text{ mH}$$

Now that the L value is known, we find R by relating the characteristic roots to α:

$$\frac{s_1 + s_2}{2} = \frac{-\alpha + \sqrt{\alpha^2 - \omega_0^2} - (\alpha - \sqrt{\alpha^2 - \omega_0^2})}{2}$$

$$\frac{s_1 + s_2}{2} = -\alpha$$

Using $\alpha = R/2L$, we have

$$R = \alpha \cdot 2L = -(s_1 + s_2)\, L = (8k + 512k)\, 250 \text{ m}\Omega$$

$$R = 130 \text{ k}\Omega$$