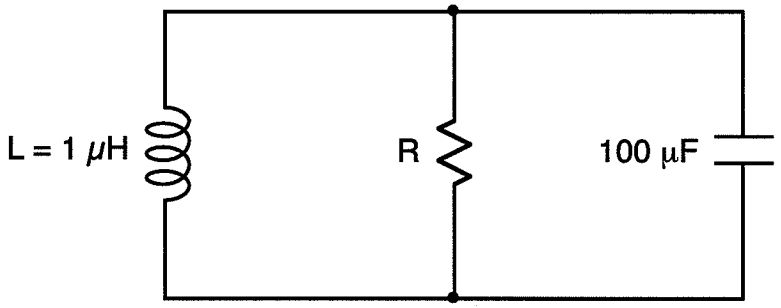
Ex:



a) If R = 40 mΩ, find the characteristic roots, s_1 and s_2, for the above circuit.

b) If R = 40 mΩ, find the value of L that makes the circuit critically damped.

c) If L = 1 μH, find the damping frequency, ω_d, for the value of R that gives a damping frequency of α = 10 krad/s.

sol'n: a) For parallel RLC, characteristic roots are

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_o^2} \qquad \alpha = \frac{1}{2RC}, \quad \omega_o^2 = \frac{1}{LC}$$

$$\alpha = \frac{1}{2 \cdot 40\,m\Omega \cdot 100\,\mu F} = \frac{1}{8k\Omega\mu} \text{ r/s} = \frac{1M}{8} \text{ r/s}$$

$$\alpha = 125\,kr/s$$

$$\omega_o^2 = \frac{1}{1\mu H \cdot 100\mu F} = \frac{(1M)^2}{10^2} (r/s)^2 = (100k\ r/s)^2$$

$$\sqrt{\alpha^2 - \omega_o^2} = \sqrt{(125k)^2 - (100k)^2} = 75k\ r/s$$

$$s_{1,2} = -125k \pm 75k\ r/s = -50k\ r/s \text{ and } -200k\ r/s$$

b) Critically damped $\Rightarrow \sqrt{\alpha^2 - \omega_o^2} = 0 \Rightarrow \alpha^2 = \omega_o^2$ $(125k)^2 = \frac{1}{LC}$

$$L = \frac{1}{(125k)^2 C} = \left(\frac{8}{1M}\right)^2 \frac{1}{100\mu} H = \frac{64\ H}{100M} = 640\,nF$$

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d) $L = 1\,\mu H$, $\alpha = 10k\ r/s$

$$\omega_d = \sqrt{\omega_0^2 - \alpha^2} \qquad \omega_0^2 = \frac{1}{LC} = \frac{1}{1\mu H \cdot 100\mu F}$$

$$\omega_0^2 = \left(\frac{1M}{10}\right)^2 = (100K)^2\ r^2/s^2$$

$$\omega_d = \sqrt{(100k)^2 - (10k)^2}\ r/s$$

$$= 10k\sqrt{10^2 - 1^2}\ r/s$$

$$= 10k\sqrt{99}\ r/s$$

$$\omega_d \approx 10k \cdot 9.5\ r/s$$

$$\omega_d \approx 99.5\ k\ r/s$$