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tool: If the output value $f(\vec{x})$ of a radial basis function network is specified for M values of $\vec{x}$ and the network has M neurons, then solving a matrix equation gives the exact values for the weights.

specified $f(\vec{x})$ values: $f(\vec{x}_1), \ldots, f(\vec{x}_M)$

$$f(\vec{x}) = \sum_{j=1}^{M} w_j r_j(\vec{x})$$

$$= w_1 r_1(\vec{x}) + \ldots + w_M r_M(\vec{x})$$

$$= r_1(\vec{x}) w_1 + \ldots + r_M(\vec{x}) w_M$$

$$= [r_1(\vec{x}), \ldots, r_M(\vec{x})] \cdot \begin{bmatrix} w_1 \\ \vdots \\ w_M \end{bmatrix}$$

$$\begin{bmatrix} f(\vec{x}_1) \\ \vdots \\ f(\vec{x}_M) \end{bmatrix} = \begin{bmatrix} r_1(\vec{x}_1), & \ldots, & r_M(\vec{x}_1) \\ \vdots & & \vdots \\ r_1(\vec{x}_M), & \ldots, & r_M(\vec{x}_M) \end{bmatrix} \begin{bmatrix} w_1 \\ \vdots \\ w_M \end{bmatrix}$$

$$\begin{bmatrix} r_1(\vec{x}_1), & \ldots, & r_M(\vec{x}_1) \\ \vdots & & \vdots \\ r_1(\vec{x}_M), & \ldots, & r_M(\vec{x}_M) \end{bmatrix}^{-1} \begin{bmatrix} f(\vec{x}_1) \\ \vdots \\ f(\vec{x}_M) \end{bmatrix} = \begin{bmatrix} w_1 \\ \vdots \\ w_M \end{bmatrix}$$

or $$R^{-1} \quad \vec{f} \quad = \quad \vec{w}$$

note: The $\vec{x}_1, \ldots, \vec{x}_M$ do <u>not</u> have to be located at the center points of the radial basis functions.

note: The values of the weights are <u>not</u> equal to the values of $f(\vec{x})$ at the center points of the radial basis functions unless the radial basis functions do not overlap each other's centers. If they don't overlap we have R^{-1} = Identity