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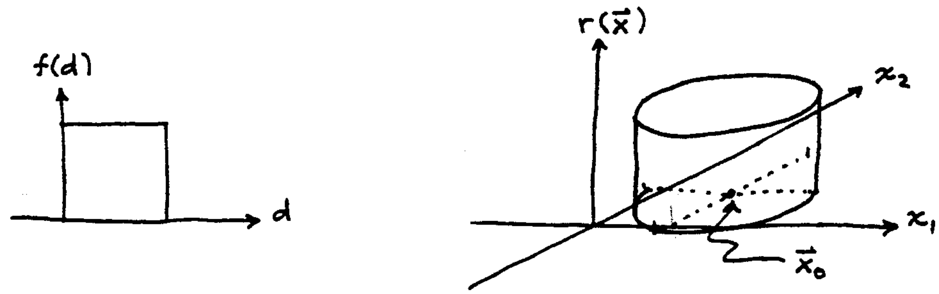
def: $r(\vec{x})$ is a radial basis function $\equiv$ the value of $r(\vec{x})$ depends only on the distance, d, of $\vec{x}$ from a center point $\vec{x}_0$. In other words, $r(\vec{x})$ has the following form:

$$r(\vec{x}) = f(d) \quad \text{and} \quad r(\vec{x}) = 1 \quad \text{when } d = 0$$

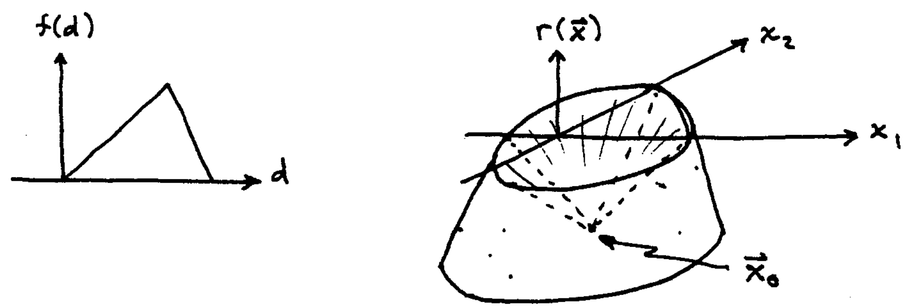
where $d = \sqrt{|\vec{x} - \vec{x}_0|^2}$ is the Euclidean distance from $\vec{x}$ to $\vec{x}_0$, ($d \geq 0$ always).

tool: We create radial basis functions by defining $f(d)$ and rotating it about a vertical axis at $\vec{x}_0$.

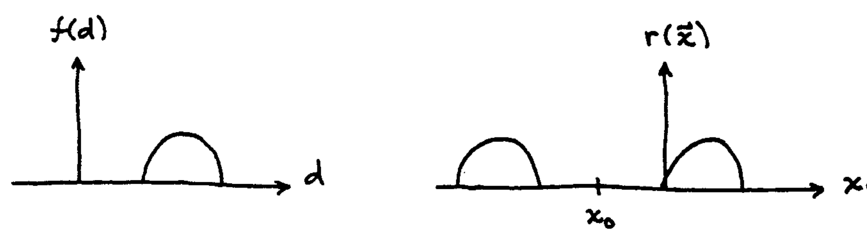
ex:



ex:



ex:



note: $\vec{x} = x_1$ (only one input variable)

note: $\vec{x} = (x_1, \ldots, x_N)$ can have more than two dimensions. We just have no way of drawing radial basis functions for $N > 2$.

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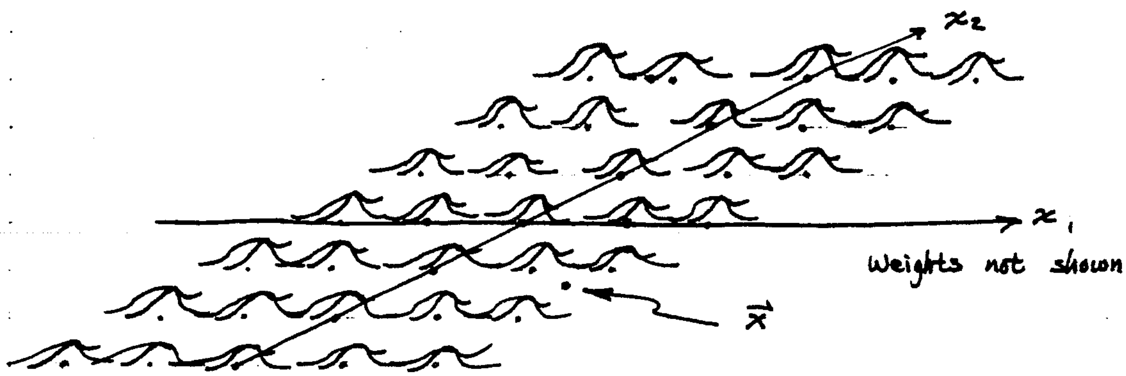
~~tool~~: We create a radial basis function network by taking a weighted sum of radial basis functions whose center points may be placed wherever we wish but are usually placed on a regular grid.

$$f(\vec{x}) = \sum_{j=1}^{M} w_j \, r_j(\vec{x})$$

where $r_j(\vec{x})$ is centered at $\vec{x}_{oj}$

~~ex~~: gaussian radial basis functions on regular grid



Each bump (basis function) has an associated weight that may be thought of as changing the height of the bump.

To evaluate $f(\vec{x})$ we determine how far $\vec{x}$ is from the center of <u>every</u> radial basis function. Having found this distance, we can determine the value of $r_j(\vec{x})$. In the picture, $r_j(\vec{x})$ is just the height of the bump $r_j(\vec{x})$ at $\vec{x}$. We note that $\vec{x}$ is quite far out on the tails of most of the gaussians. Hence, $r_j(\vec{x})$ is small for most of the r_j's.

Given the $r_j(x)$ values, multiply by weights w_j and sum to get the network output.

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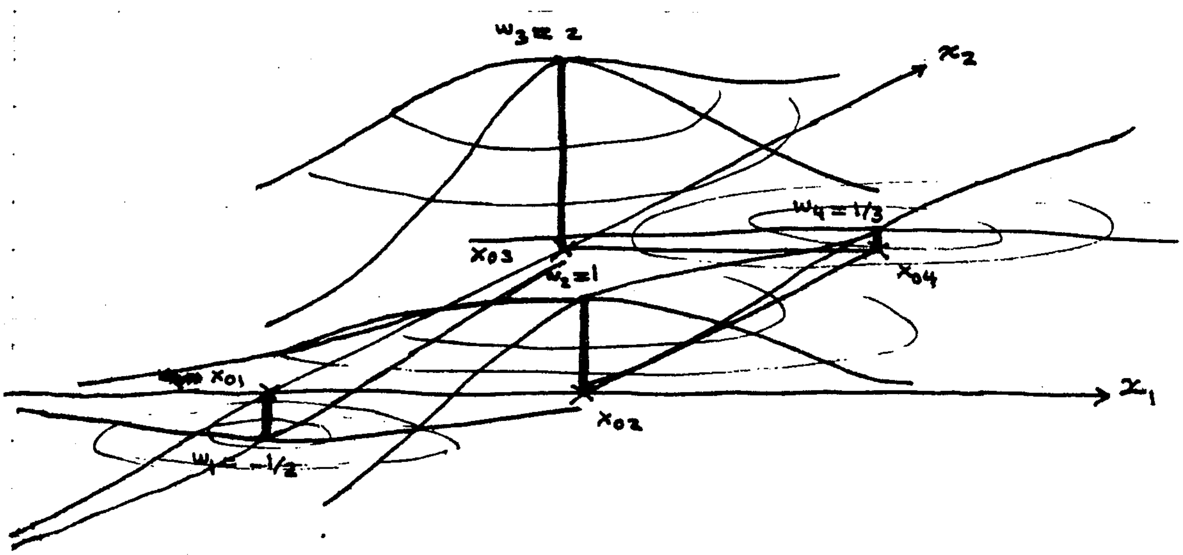
ex: Network with four gaussian radial basis functions, weights indicated by heights.

$$r_1(\vec{x}) = e^{-(x_1^2 + x_2^2)} \qquad r_2(\vec{x}) = e^{-[(1-x_1)^2 + x_2^2]}$$

$$r_3(\vec{x}) = e^{-[x_1^2 + (1-x_2)^2]} \qquad r_4(\vec{x}) = e^{-[(1-x_1)^2 + (1-x_2)^2]}$$

$$w_1 = -\tfrac{1}{2} \qquad w_2 = 1 \qquad x_{01} = (0,0) \qquad x_{02} = (1,0)$$

$$w_3 = 2 \qquad w_4 = \tfrac{1}{3} \qquad x_{03} = (0,1) \qquad x_{04} = (1,1)$$



ex: Evaluate $f(\vec{x})$ for above example with $\vec{x} = (\tfrac{1}{2}, \tfrac{1}{2})$

$$r_1(\vec{x}) = e^{-[(\frac{1}{2})^2 + (\frac{1}{2})^2]} = e^{-\frac{1}{2}} = 0.606$$

$$r_2(\vec{x}) = e^{-[(1-\frac{1}{2})^2 + (\frac{1}{2})^2]} = e^{-\frac{1}{2}} = 0.606$$

$$r_3(\vec{x}) = e^{-[(\frac{1}{2})^2 + (1-\frac{1}{2})^2]} = e^{-1/2} = 0.606$$

$$r_4(\vec{x}) = e^{-[(1-\frac{1}{2})^2 + (1-\frac{1}{2})^2]} = e^{-\frac{1}{2}} = 0.606$$

$$f(\vec{x}) = \sum_{j=1}^{4} w_j\, r_j(\vec{x}) = -\tfrac{1}{2} \cdot 0.606 + 1 \cdot 0.606 + 2 \cdot 0.606 + \tfrac{1}{3} 0.606$$

$$= 1.7$$

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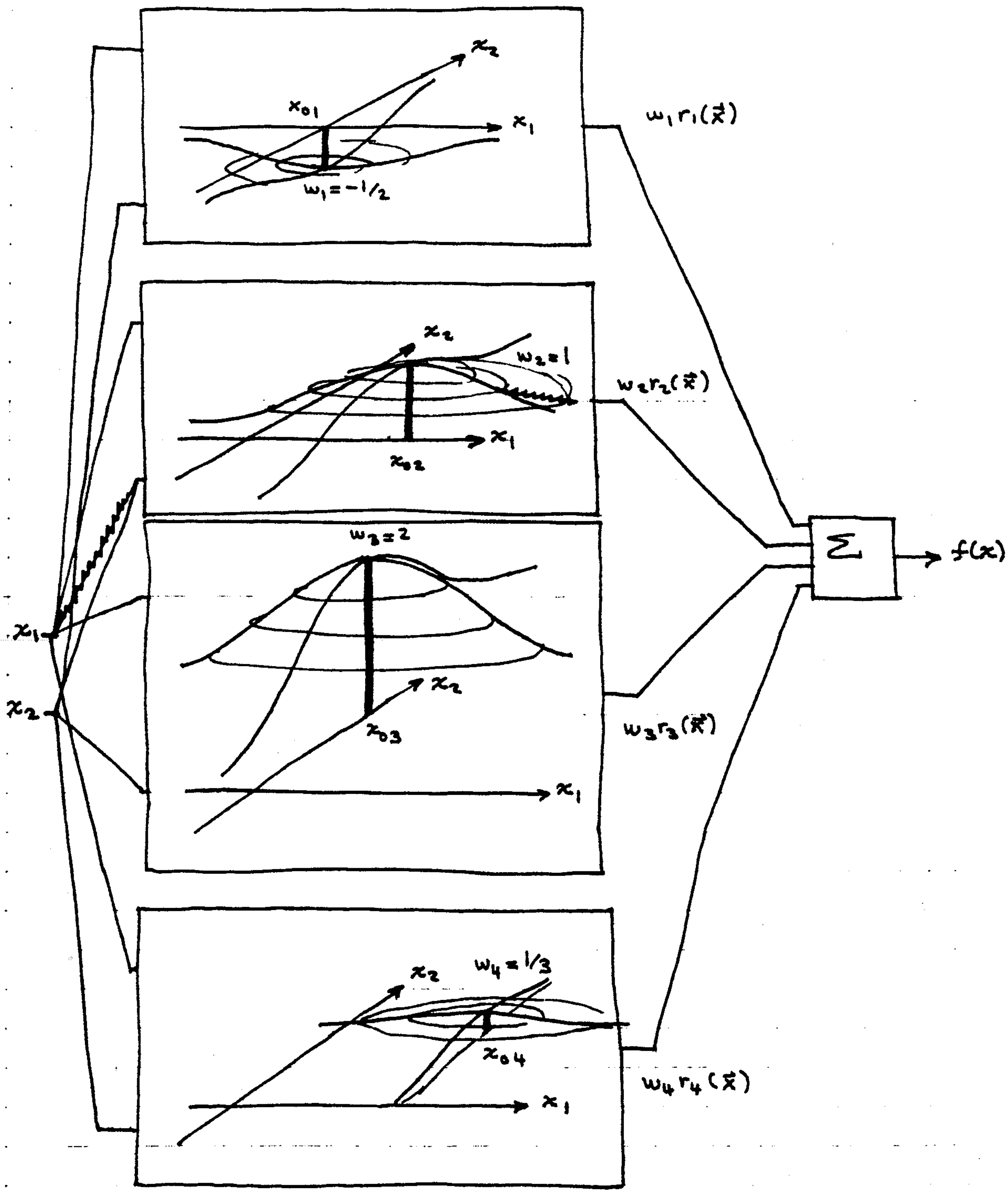
ex: We can draw the previous example network as follows.



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ex: We can also draw the previous network example as follows.