May 1990 Real Analysis — Convergence ~~[illegible]~~ — Uniformly

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$\langle f_n \rangle \equiv$ sequence of functions on domain D

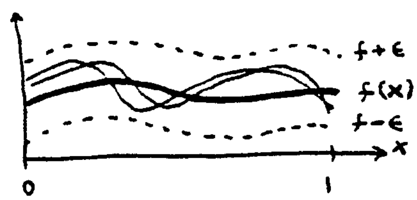
$\langle f_n \rangle$ converges to f uniformly $\equiv$

Given $\epsilon > 0$, there is an N such that

$|f_n(x) - f(x)| < \epsilon$ for all x and for all $n \geq N$.

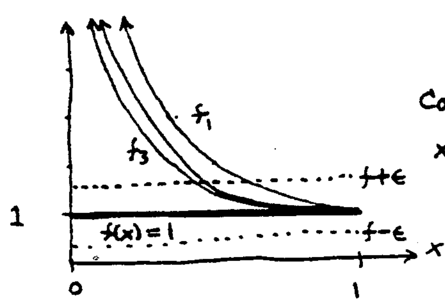
The idea is that if we take an ϵ envelope around $f(x)$ we can find N such that every f_n lies in that ϵ envelope for $n \geq N$ (i.e. beyond a certain point in the sequence).

picture:



Intuitively, uniform convergence guarantees that the rate of convergence is not too slow at some x.

ex: $\left\langle f_n(x) = \frac{1}{x^{1/n}} \right\rangle \rightarrow f(x) = 1$ pointwise but not uniformly on $(0, 1]$



Converges too slowly for x near zero. For any ϵ envelope, there is an x near zero such that $f_n(x)$ will lie outside the envelope for n arbitrarily large.

ex: $\left\langle f_n(x) = \frac{1}{n} \right\rangle \rightarrow f(x) = 0$ uniformly on $[0,1]$